Random Coupling Model for Wireless Communication Channels

Gabriele Gradoni*†, Xiaoming Chen‡, Thomas M. Antonsen†, Steven M. Anlage§, and Edward Ott†

*School of Mathematical Sciences, University of Nottingham
Nottingham NG7 2RD, United Kingdom Email: gabriele.gradoni@nottingham.ac.uk
†Institute for Research in Electronics and Applied Physics, University of Maryland
College Park, Maryland 20742, Email: {ggradoni, antonsen, edott}@umd.edu
‡Department of Signals and Systems, Chalmers University of Technology
Gothenburg, Sweden, Email: xiaoming.chen@chalmers.se
§Center for Nanophysics & Advanced Materials, University of Maryland
College Park, Maryland 20742, Email: anlage@umd.edu

***Abstract*—In this paper, we derive an explicit formula for the voltage-to-voltage transfer function of multiple-input multiple-output (MIMO) wireless channels. A statistical model, the random coupling model, is used to develop the open-circuit transfer function of the MIMO channel on a physical basis. The emulation of realistic wireless channels is typically performed through irregular cavities with high losses. In this case, we find that the transfer function takes a simple form involving the free-space impedance matrix of antennas and a fluctuation matrix expressing the wave chaos inside the environment. Monte Carlo simulations of the open-circuit transfer function are performed for MIMO systems up to three antennas in the transmit and receive arrays. In contrast to the common assumption that the MIMO channel fulfill multivariate normality (MVN), the Hans-Zinckler test of the obtained ensembles of the MIMO channel show that the MVN assumption of the MIMO channel tends to be invalid with an increasing number of antennas in the transmitting and receiving arrays, when mutual coupling is present in the arrays. Numerical results indicate that this effect is more pronounced at relatively low frequencies.**

***Index Terms*—statistical electromagnetics; reverberation chamber; losses; wireless channel; MIMO; multivariate normality; random matrix theory; wave chaos**

I. INTRODUCTION

Over the last decade, the electromagnetic reverberation chamber (RC) has been used as an efficient emulator for rich fading environments [1], [2], [3]. Recent telecommunication systems rely on multi-antenna techniques to achieve higher data rates (throughputs) or more reliable wireless connectivity. Striking examples are the long-term evolution (LTE) [4] and the wireless local area network (WLAN) system [5], where the spatial domain is exploited in order to improve the throughput (via spatial multiplexing) and/or the reliability (via spatial/polarization diversity) in a so-called multiple-input multiple-output (MIMO) system [6].

Laboratory framework for the emulation of those wireless MIMO channels are typically made of a transmitting and a receiving antenna array, both operating inside shielded rooms. In order to make those rooms close to real-life environments, the metallic boundaries are intentionally made irregular through stirring structures. This also serves as a mean to create enough independent samples [7]. In order to control the delay spread of the channel [8], artificial losses are introduced in the room through absorbing materials and openings [9]. Given the complexity of such an environment, a statistical descriptions becomes appropriate due to the facts that the fields are mixed up, and the boundary geometry is hard-to-model either with closed mathematical formula or with detailed numerical representations. In the statistical point-of-view, the cavity eigenspectrum can be reproduced by universal statistical laws of the random matrix theory, and related eigenmode amplitudes are locally replaced by a random superposition of plane-waves being mode topologies speckled and highly delocalized. The random coupling model (RCM) takes those two prescriptions into the exact (deterministic) model of the cavity impedance matrix [10]. As a result, the free-space radiation impedance of the antennas can be easily de-embedded from the chaotic scattering due to irregular modes. The RCM has been recently applied to model fading of fields propagating through a random environment [11], [12].

Recently, a deviation of the MIMO channel from multivariate normality (MVN) has been demonstrated by emulating it in an RC [13]. However, the flexibility of the empirical verifications [13] is limited by the practical measurement setup (e.g., fixed transmitting antennas). The purpose of this work is to use the random coupling model (RCM) to further investigate the MVN property of the MIMO channel. Once the free-space impedances of the antennas are determined (by measurements of the real antennas in an anechoic chamber or simulations of the modeled antennas in a full-wave simulation software), one can use the RCM with great flexibility to create ensembles of wireless MIMO channels with arbitrary losses and number of antennas.

II. TRANSFER FUNCTION

In order to characterize the wireless channel response, it is important to establish a relation between antenna port voltages and currents. This is typically represented as a (multi-port) impedance matrix of the antennas radiating inside the environment (cavity), namely $\underline{\underline{Z}}^{cav}$, defined as

$$\underline{V} = \underline{\underline{Z}}^{cav}(\alpha)\,\underline{I} \ . \tag{1}$$

If we separate transmitting and receiving antennas, the impedance transfer function takes the form

$$\begin{bmatrix} \underline{V}_T \\ \underline{V}_R \end{bmatrix} = \begin{bmatrix} \underline{\underline{Z}}_T & \underline{\underline{H}}_{oc} \\ \underline{\underline{H}}_{oc} & \underline{\underline{Z}}_R \end{bmatrix} \begin{bmatrix} \underline{I}_T \\ \underline{I}_R \end{bmatrix} \ , \tag{2}$$

where $\underline{\underline{Z}}_T$ and $\underline{\underline{Z}}_R$ are the impedance matrices of the transmitting (T) and the receiving (R) antenna arrays respectively, with $\underline{V}_T$, $\underline{V}_R$, $\underline{I}_T$, $\underline{I}_R$ port voltages and currents of the transmitting and receiving antenna arrays respectively. It is worth pointing out that, in irregular cavities, the impedance transfer function in (1) and (2) are populated by random variables. The statistical approach used to generate those random variables will be discussed in section III. After some algebraic manipulations, we obtain an explicit relation between the set of voltages at the receiving antennas and the set of voltage at the transmitting antennas, namely, the voltage-voltage transfer function $\underline{\underline{H}}_V$. In particular, by assuming that the receiving array does not feedback signals to the transmitting array, and that there is a very large attenuation between the two arrays, i.e., for small fading fluctuations of $\underline{\underline{H}}_{oc}$, we can use the unilateral approximation of the circuit theory for communications [14]

$$\underline{V}_R = \underline{\underline{H}}_V \underline{V}_T \ , \tag{3}$$

where $\underline{\underline{H}}_V$ is found to be well approximated by

$$\underline{\underline{H}}_V \approx \underline{\underline{Z}}_L \left(\underline{\underline{Z}}_L + \underline{\underline{Z}}_R\right)^{-1} \underline{\underline{H}}_{oc} \left(\underline{\underline{Z}}_T + \underline{\underline{Z}}_S\right)^{-1} \ , \tag{4}$$

where $\underline{\underline{H}}_{oc}$ represents the open circuit transfer function between port currents at the transmit array and port voltages at the receive array. We will see later on the paper that the unilateral approximation is self-consistent with the realm of wireless environments, where the amount of losses is very high on account of the proper delay spread. It is worth noticing that the right part of (4) expresses the voltage-to-voltage partitioning at the load, and the unique connection between the transmitter and the receiver is established by $\underline{\underline{H}}_{oc}$, under the assumption that the receiving antennas are open-circuited. Also, we notice that $\underline{\underline{H}}_V$ is a version of the open-circuit transfer function normalized to antenna impedance matrices. The information theoretic channel transfer function is also a renormalized version of the open-circuit channel transfer function [14]

$$\underline{\underline{H}}_{eff} = \sqrt{N_T}\left[\Re\left(\underline{\underline{Z}}_L\right)\right]^{1/2} \left(\underline{\underline{Z}}_L + \underline{\underline{Z}}_R\right)^{-1} \underline{\underline{H}}_{oc} \left[\Re\left(\underline{\underline{Z}}_T\right)\right]^{-1/2} . \tag{5}$$

In the absence of a direct line-of-sight (LOS), the only physical way for the wireless signal to get from the transmitter to the receiver is through multiple reflection and scattering. In an RC where no clear LOS path exists, it is assumed that the transmitter and the receiver are coupled only through cavity modes. Therefore, for transceivers in a non-LOS real-life (highly irregular) wireless environment, $\underline{\underline{H}}_{oc}$ is well approximated by the transfer impedance of a pure chaotic cavity [15]. In high-loss regime, asymptotic statistics of both the real and the imaginary parts of the elements of the cavity impedance matrices are expected to be zero mean Gaussian distributed random variables [16].

III. Random Coupling Model

The RCM is suitable for modeling the wireless channel in an irregular cavity with arbitrary losses. In particular, we infer that the boundary structure is irregular enough to create wave mixing and chaos, for which the statistics of the cavity eigenmodes can be reproduced and predicted from the universal laws of the random matrix theory (RMT). Inherently, the general multi-port RCM can be used to model the impedances of transmitting and receiving arrays radiating inside the cavity [10]

$$\begin{aligned} \underline{\underline{Z}}_R &= j\Im\left(\underline{\underline{Z}}_R^{rad}\right) + \\ &\left[\Re\left(\underline{\underline{Z}}_R^{rad}\right)\right]^{1/2} \cdot \underline{\underline{\xi}}^{cav}(\alpha)\left[\Re\left(\underline{\underline{Z}}_R^{rad}\right)\right]^{1/2} \ , \\ \underline{\underline{Z}}_T &= j\Im\left(\underline{\underline{Z}}_T^{rad}\right) + \\ &\left[\Re\left(\underline{\underline{Z}}_T^{rad}\right)\right]^{1/2} \cdot \underline{\underline{\xi}}^{cav}(\alpha)\left[\Re\left(\underline{\underline{Z}}_T^{rad}\right)\right]^{1/2} \ , \end{aligned} \tag{6}$$

where the impedance matrix $\underline{\underline{Z}}_{T,R}^{rad}$ represents the free-space radiation impedances of transmitting and receiving antennas, which can be measured in anechoic chamber. Interestingly, in case no LOS is present between transmitting and receiving antenna, $\underline{\underline{Z}}_T^{rad}$ and $\underline{\underline{Z}}_R^{rad}$ can be kept separate as shown in (2). Consequently, since the open-circuit transfer function couples the set of voltages at the open-circuited receiving antennas with the set of currents at the transmitting antennas, RCM suggests that in high frequency regime, where the cavity exhibit pure wave chaos [16]

$$\underline{\underline{H}}_{oc} \approx \left[\Re\left(\underline{\underline{Z}}_R^{rad}\right)\right]^{1/2} \cdot \underline{\underline{\xi}}^{cav}(\alpha)\left[\Re\left(\underline{\underline{Z}}_T^{rad}\right)\right]^{1/2} \ , \tag{7}$$

where α is the loss factor of the closed environment. In (7), the fluctuation matrix $\underline{\underline{\xi}}^{cav}$

$$\underline{\underline{\xi}}^{cav} = \frac{-j}{\pi}\sum_n \frac{\underline{\Phi}_n \underline{\Phi}_n^T}{\mathcal{K}_0 - \mathcal{K}_n + i\alpha} \ , \tag{8}$$

is the sum over irregular modes, where $\underline{\Phi}_n$ is a column vector of zero-mean Gaussian random variables representing the (random) coupling between antennas and cavity modes, $\mathcal{K}_0 = k_0^2/\Delta k^2$, and $\mathcal{K}_n^2 == k_n^2/\Delta k^2$ are Wigner distributed (chaotic) eigen-energies of the unperturbed cavity, which can be reproduced by eigenvalues of large random matrices of the Gaussian orthogonal ensemble (GOE) [17].

The $\underline{\underline{\xi}}^{cav}$ matrix is driven by universal laws of RMT, and it depends on a unique scalar parameter [10]

$$\alpha = \frac{k_0^2}{Q\,\Delta k^2} \ , \tag{9}$$

where the average quality factor of the unperturbed cavity can be calculated as

$$Q = \frac{3V}{2\delta\mu_r S} \tag{10}$$

and the average nearest neighboring mode spacing is given by, for a 3D cavity [18]

$$\Delta k^2 = \frac{2\pi^2}{V k_0^2} \; . \tag{11}$$

Interestingly, in high-loss limit, the small fluctuation approximation can be invoked in the RCM [15], for which (4) can be written as

$$\underline{\underline{H}}_V \approx \underline{\underline{Z}}_L \left(\underline{\underline{Z}}_L + \underline{\underline{Z}}_R^{rad}\right)^{-1} \underline{\underline{H}}_{oc} \left(\underline{\underline{Z}}_T^{rad} + \underline{\underline{Z}}_S\right)^{-1} \; , \tag{12}$$

where now there is only one fluctuating matrix, $\underline{\underline{H}}_{oc}$, predicted by (7).

It is important to notice that (7) gives a physical model in alternative to the correlation-based statistical models such as the Kroneker model [6]

$$\underline{\underline{H}}_{oc} = \left[\underline{\underline{R}}_T\right]^{1/2} \underline{\underline{H}}_w \left[\underline{\underline{R}}_R\right]^{1/2} \; , \tag{13}$$

or the full-correlation model [6]

$$\underline{\underline{H}}_{oc} = \left[\underline{\underline{R}}\right]^{1/2} \underline{\underline{H}}_w \; , \tag{14}$$

where $\underline{\underline{R}}_T$ and $\underline{\underline{R}}_R$ are Kronecker partitions of the full correlation matrix $\underline{\underline{R}}$, and $\underline{\underline{H}}_w$ is a channel transfer matrix with $N_T \times N_R$ independent and identically distributed (i.i.d.) zero mean Gaussian random variables.

A Monte Carlo analysis of (7) can be performed by generating an ensemble of cavity responses using the method described in [19].

IV. Monte Carlo generation

We generate ensembles of random matrices for the MIMO channel transfer function (7) of an RC equipped with an array of 3 transmitting antennas, a circular array of 3 receiving antenna elements, and absorbing materials in order to achieve a certain delay spread, and hence to emulate actual wireless environments. The average quality factor [20] of the adopted RC [21] is then very low, $Q = 300$ at 900 MHz. This corresponds to a suboptimal operation of the RC in the perspective of electromagnetic compatibility (EMC) studies [22]. The chamber volume is $1.75 \times 1.25 \times 1.8$ m^3. We assume identical transmitting and receiving antenna arrays, having impedance matrix $\underline{\underline{Z}}_R^{rad} = \underline{\underline{Z}}_T^{rad}$, and terminated to load impedances $\underline{\underline{Z}}_L^{rad} = 50\ \Omega \times \underline{\underline{I}}$, with $\underline{\underline{I}}$ identity matrix.

At low losses, the fluctuation of the elements of the transfer matrix are intrinsically non-Gaussian [11]. This is also true at relatively low frequencies, below the lowest usable frequency (LUF) of the chamber, within the so-called undermoded regime [23], [24], [25].

At high losses, which is $\alpha > 6$, the fluctuation of the elements of the channel transfer matrix is expected to be zero-mean Gaussian distributed for both the real and the imaginary parts. We begin with the basic configuration having one transmitting and one receiving antenna with real input impedance of 50 Ω. Equation (7) becomes a scalar random variable, whose statistics can be predicted by generating ensembles of chaotic cavity responses (8) as described in the previous section and detailed in [26]. In this application, we generate 1000 random cavity realizations, each one given by the superposition of 1000 random modes. Also, in order to emulate realistic scenarios, we adopt a value of 6.6 for the loss factor α, being real-life wireless channels characterized by several absorption mechanisms and power leakage.

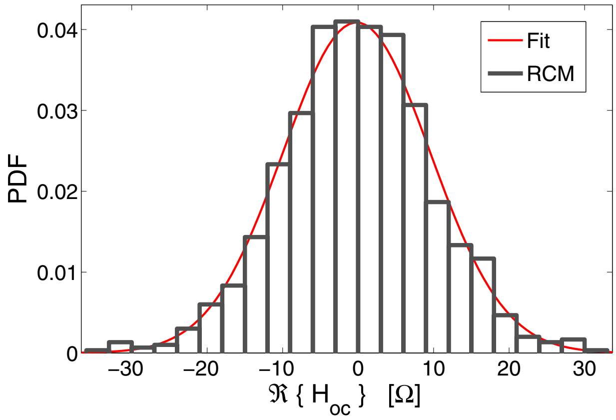

Fig. 1. Probability density function of the real part of a single-input single-output (SISO) wave chaotic environment with high losses, such as the loaded reverberation chamber adopted to emulate the wireless electromagnetic environment. Monte Carlo generation of (7) based on the random coupling model result in a density function (grey bars) which is well fitted with a normal distribution (red solid line) at 900 MHz.

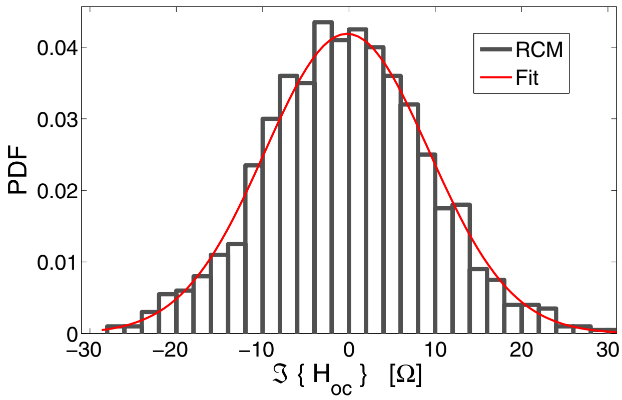

Fig. 2. Probability density function of the real part of a single-input single-output (SISO) wave chaotic environment with high losses, such as the loaded reverberation chamber adopted to emulate the wireless electromagnetic environment. Monte Carlo generation of (7) based on the random coupling model result in a density function (grey bars) which is well fitted with a normal distribution (red solid line) at 900 MHz.

Figs. (1) and (2) show the probability density function of the real and imaginary part of the channel transfer matrix, as generated from (7) through the RCM, for the modeled RC excited at 900 MHz. A very good fitting from a normal distribution is obtained. We then increase the number of transmitting and receiving antennas to two and three antenna elements for each array. Inherently, the mutivariate normality of all the elements of the channel (12) can be tested through the Hans-Ziekler test [27]. The outcome of this test is a p-value expressing the acceptance probability against a confidence interval β, which is typically set to 0.05. The Monte Carlo generation of $\underline{\underline{H}}_{oc}$ based on the RCM with uncoupled antennas in both the arrays, i.e., diagonal $\underline{\underline{Z}}_T$ and $\underline{\underline{Z}}_R$, gives a highly fluctuating p-value for two and three radiating elements. This outcome is not stable in the sense that it looks to take values lower or higher than β with a uniform probability, hence not suggesting a clear conclusion on MVN of the process.

We then push this study ahead by introducing an artificial mutual coupling within the antenna elements in the transmit and receive arrays. This is suggested by previous studies in reverberation chamber [8]. In particular, we assume non-diagonal free-space impedance matrices $\underline{\underline{Z}}_T$ and $\underline{\underline{Z}}_R$. This is physically consistent with the setup adopted to emulate MIMO channels inside an RC, where spatial/polarization diversity is realized through antenna arrays with radiating elements that are confined in a small space and thus put in close proximity, i.e., at distances below the wavelength of the RC excitation. The numerical values adopted in our Monte Carlo simulations are

$$\underline{\underline{Z}}_T = \underline{\underline{Z}}_R = \begin{bmatrix} 50 & 20-j30 & 20-j30 \\ 20-j30 & 50 & 20-j30 \\ 20-j30 & 20-j30 & 50 \end{bmatrix} \Omega\ . \quad (15)$$

For $N = 1$ antenna we have $Z_t = Z_R = 50\ \Omega$, while for $N = 2$ antennas we have the 2×2 partition obtained by excluding the third raw and the third column in (15).

The obtained p-values in presence of mutual coupling are reported in Tab. I. The results denote a clear degradation

TABLE I
p-VALUES OF THE HANS-ZINCKLER MULTIVARIATE NORMALITY TEST OF (12) (CONFIDENCE INTERVAL $\beta = 0.05$) FOR $N = 1$, $N = 2$, AND $N = 3$ ANTENNAS IN BOTH THE TRANSMIT AND THE RECEIVE ARRAYS, AT SELECTED FREQUENCIES OF INTEREST IN MODERN TELECOMMUNICATIONS, WHICH ARE 800, 900, AND 1000 MHZ.

	$N=1$	$N=2$	$N=3$
800 MHz	0.3762997	0.0002569	0.0000000
900 MHz	0.0679909	0.0000008	0.0000000
1000 MHz	0.7668212	0.0001664	0.0000000

of the MVN with the increasing number of antennas in the transmitting or(and) receiving array(s). One further numerical observation is that the rejection rate is now stable in terms of the condition $p < \beta$ for $N \geq 2$, i.e., it does not fluctuate with the generation of ensembles of MIMO channels. We further explore MVN degradation in a broad frequency range. The results are reported in Fig. 3 for a single-input single-output system, in Fig. 4 for a double-input double-output system, and in Fig. 5 for a triple-input triple-output system. In these Monte Carlo simulations we used 200 cavity realizations, each one made of 1000 ergodic cavity modes. The sought degradation becomes clear at relatively low frequencies, for an increasing number of antennas in the arrays. This is a numerical confirmation of previously observed behaviors of MIMO channels [13], given in terms of a first-principle physical model. The trend followed by the MVN degradation is confirmed by repeating the test at different values of the loss

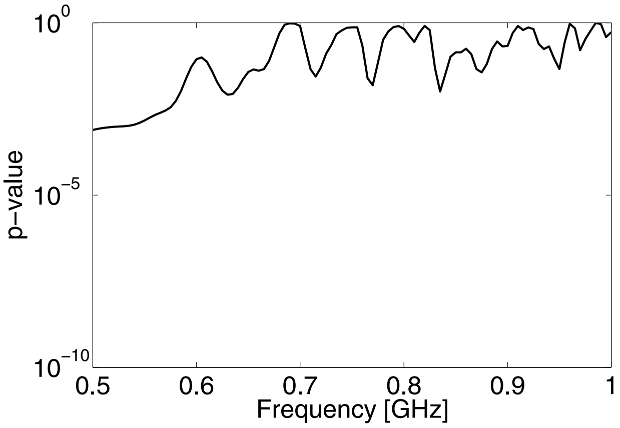

Fig. 3. Hans-Zinckler multivariate normality test of (12) (confidence interval $\beta = 0.05$) for a single-input single-output system. p-values are reported in the frequency range from 0.5 GHz to 1 GHz.

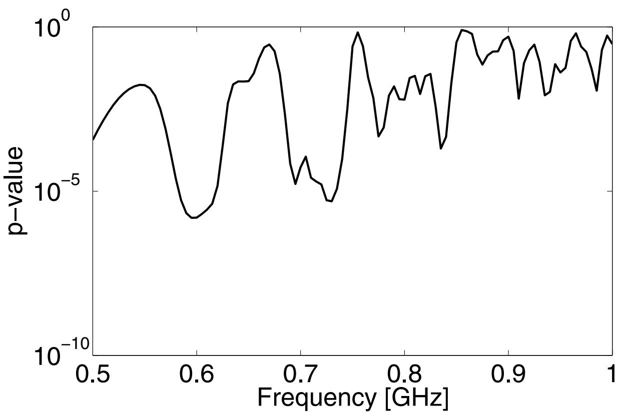

Fig. 4. Hans-Zinckler multivariate normality test of (12) (confidence interval $\beta = 0.05$) for a double-input double-output system. p-values are reported in the frequency range from 0.5 GHz to 1 GHz.

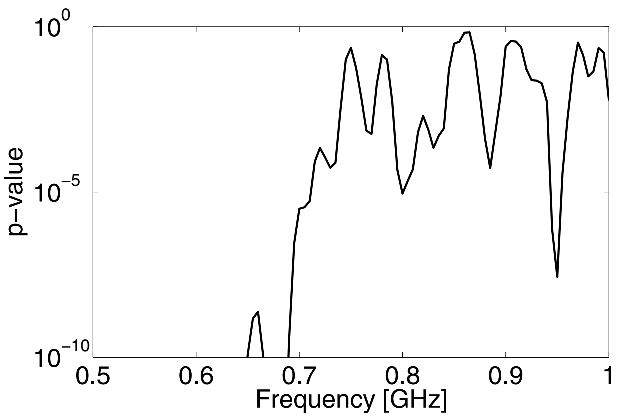

Fig. 5. Hans-Zinckler multivariate normality test of (12) (confidence interval $\beta = 0.05$) for a triple-input triple-output system. p-values are reported in the frequency range from 0.5 GHz to 1 GHz.

factor (α) and of the elements of the free-space impedance matrices. The presence of losses along with the correlation between antenna elements are not accounted for properly in the models (13) and (14), commonly accepted to reproduce the statistics of MIMO channels. The non-MVN deserves further investigation in order to establish the conditions for which the normality can be restored or avoided even in the presence of losses and correlation between antennas.

V. Conclusions

A first principle model of the voltage-to-voltage transfer function has been derived for MIMO systems operating in a lossy and irregular electromagnetic environment. The model relies on a statistical representation of the environment based on wave chaos, which gives the impedance matrices of antenna arrays radiating inside a cavity in terms of their free-space impedance matrices. In contrast to the common assumption that the elements of a MIMO channel matrix is jointly Guassian (MVN), it is found that in the presence of losses and moderate frequencies with respect to the chamber size, the likelihood of the MVN of the channel matrix decreases with the increasing number of antenna elements and in in presence of mutual coupling in the transmitting and/or receiving arrays. This effect shows up clearly at relatively low frequencies. This means more attention should be paid in stochastic modeling of MIMO channels and theoretical study of MIMO systems. This work also serves a feasibility study of using RCM to investigate the MVN of MIMO channels. The agreement of the results in this paper with the previous RC measurements implies that the RCM can be used conveniently to study the MVN of various MIMO channels with arbitrary transmitting and receiving arrays.

Acknowledgement

The authors wish to acknowledge the members of the Maryland Wave Chaos group for their helpful comments. G. G.; T. M. A.; S. M. A.; and E. O. acknowledge the financial support of the Air Force Office of Scientific Research FA95501010106 and the Office of Naval Research N00014130474.

References

[1] E. Genender, C. Holloway, K. Remley, J. Ladbury, G. Koepke, and H. Garbe, "Simulating the multipath channel with a reverberation chamber: Application to bit error rate measurements," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 52, no. 4, pp. 766–777, 2010.

[2] M. Garcia-Fernandez, J. Sanchez-Heredia, A. Martinez-Gonzalez, D. Sanchez-Hernandez, and J. Valenzuela-Valdes, "Advances in mode-stirred reverberation chambers for wireless communication performance evaluation," *Communications Magazine, IEEE*, vol. 49, no. 7, pp. 140–147, 2011.

[3] X. Chen, P.-S. Kildal, and M. Gustafsson, "Characterization of implemented algorithm for MIMO spatial multiplexing in reverberation chamber," *Antennas and Propagation, IEEE Transactions on*, vol. 61, no. 8, pp. 4400–4404, Aug 2013.

[4] X. Chen, "Throughput modeling and measurement in an isotropic-scattering reverberation chamber," *Antennas and Propagation, IEEE Transactions on*, vol. 62, no. 4, pp. 2130–2139, April 2014.

[5] R. Recanatini, F. Moglie, and V. Primiani, "Performance and immunity evaluation of complete WLAN systems in a large reverberation chamber," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 55, no. 5, pp. 806–815, 2013.

[6] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communication*. Cambridge University Press, 2003.

[7] L. Arnaut, "Effect of local stir and spatial averaging on measurement and testing in mode-tuned and mode-stirred reverberation chambers," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 43, no. 3, pp. 305–325, 2001.

[8] X. Chen, P. S. Kildal, C. Orlenius, and J. Carlsson, "Channel sounding of loaded reverberation chamber for over-the-air testing of wireless devices: Coherence bandwidth versus average mode bandwidth and delay spread," *Antennas and Wireless Propagation Letters, IEEE*, vol. 8, pp. 678–681, 2009.

[9] D. A. Hill, M. T. Ma, A. R. Ondrejka, B. F. Riddle, M. L. Crawford, and R. T. Johnk, "Aperture excitation of electrically large, lossy cavities," *IEEE Trans. Electromagn. Compat.*, vol. 36, no. 3, pp. 169–178, Aug. 1994.

[10] G. Gradoni, J.-H. Yeh, B. Xiao, T. M. Antonsen, S. M. Anlage, and E. Ott, "Predicting the statistics of wave transport through chaotic cavities by the random coupling model: A review and recent progress," *Wave Motion*, vol. 51, no. 4, pp. 606 – 621, 2014, innovations in Wave Modelling.

[11] J.-H. Yeh, T. M. Antonsen, E. Ott, and S. M. Anlage, "First-principles model of time-dependent variations in transmission through a fluctuating scattering environment," *Phys. Rev. E(Rapid Communications)*, vol. 85, p. 015202, 2012.

[12] J.-H. Yeh, E. Ott, T. M. Antonsen, and S. M. Anlage, "Fading statistics in communications - a random matrix approach," *Acta Physica Polonica A*, vol. A-85, p. 120, 2012.

[13] X. Chen, "Spatial correlation and ergodic capacity of MIMO channel in reverberation chamber," *International Journal of Antennas and Propagation*, vol. 2012, pp. 1–7, 2012.

[14] M. Ivrlac and J. Nossek, "Toward a circuit theory of communication," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 57, no. 7, pp. 1663–1683, 2010.

[15] G. Gradoni, T. M. Antonsen, and E. Ott, "Impedance and power fluctuations in linear chains of coupled wave chaotic cavities," *Phys. Rev. E*, vol. 86, p. 046204, Oct 2012.

[16] X. Zheng, T. M. Antonsen, and E. Ott, "Statistics of impedance and scattering matrices of chaotic microwave cavities with multiple ports," *Electromagnetics*, vol. 26, p. 37, 2006.

[17] M. L. Mehta, *Random Matrices*, 3rd ed., ser. Pure and Applied Mathematics. San Diego, CA: Elsevier Science, 2004.

[18] U. Dörr, H.-J. Stöckmann, M. Barth, and U. Kuhl, "Scarred and chaotic field distributions in a three-dimensional sinai-microwave resonator," *Phys. Rev. Lett.*, vol. 80, pp. 1030–1033, Feb 1998.

[19] S. Hemmady, X. Zheng, T. M. Antonsen, E. Ott, and S. M. Anlage, "Universal statistics of the scattering coefficient of chaotic microwave cavities," *Phys. Rev. E*, vol. 71, p. 056215, May 2005.

[20] P. Corona, G. Ferrara, and M. Migliaccio, "A spectral approach for the determination of the reverberating chamber quality factor," *IEEE Trans. Electromagn. Compat.*, vol. 40, no. 2, pp. 145–153, 1998.

[21] X. Chen, "On independent platform sample number for reverberation chamber measurements," *IEEE Trans. Electromagn. Compat.*, vol. 54, no. 6, pp. 1306–1309, Dec. 2012.

[22] C. L. Holloway, D. A. Hill, J. M. Ladbury, and G. Koepke, "Requirements for an effective reverberation chamber: Unloaded or loaded," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 1, pp. 187–194, Feb. 2006.

[23] G. Orjubin, E. Richalot, S. Mengue, and O. Picon, "Statistical model of an undermoded reverberation chamber," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 48, no. 1, pp. 248 –251, feb. 2006.

[24] C. Lemoine, P. Besnier, and M. Drissi, "Estimating the effective sample size to select independent measurements in a reverberation chamber," *IEEE Trans. Electromagn. Compat.*, vol. 50, no. 2, pp. 227–236, 2008.

[25] L. R. Arnaut, "Optimizing low-frequency mode stirring performance using principal component analysis," *IEEE Trans. Electromagn. Compat.*, 2014, to be published, DOI: 10.1109/TEMC.2013.2271903.

[26] S. Hemmady, T. Antonsen, E. Ott, and S. Anlage, "Statistical prediction and measurement of induced voltages on components within complicated enclosures: A wave-chaotic approach," *IEEE Trans. Electromagn. Compat.*, vol. 54, no. 4, pp. 758–771, 2012.

[27] N. Henze and B. Zirkler, "A class of invariant consistent tests for multivariate normality," *Communications in Statistics Theory and Methods*, vol. 19, no. 10, pp. 3595–3618, 1990.