Analog Experiments on Quantum Chaotic Scattering and Transport

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Quantum Transport

Mesoscopic and Nanoscopic systems show quantum effects in transport:
- Conductance ~ e²/h per channel
- Wave interference effects
- “Universal” statistical properties

However these effects are partially hidden by finite-temperatures, electron de-phasing, and electron-electron interactions.

Also theory calculates many quantities that are difficult to observe experimentally, e.g.
- scattering matrix elements
- complex wavefunctions
- correlation functions

Develop a simpler experiment that demonstrates the wave properties without all of the complications
  ► Electromagnetic resonator
Outline

• What is Wave Chaos?

• Statistical Properties of Wave Chaotic Systems

• Our Microwave Analog Experiment

• One Experimental Result: Universal Conductance Fluctuations

• Ongoing Work

• Conclusions
Wave Chaos?

1) Classical chaotic systems have diverging trajectories

2-Dimensional “billiard” tables with hard wall boundaries

Regular system

Chaotic system

2) Linear wave systems can’t be chaotic

In the ray-limit it is possible to define chaos

“ray chaos”

3) However in the semiclassical limit, you can think about rays

Wave Chaos concerns solutions of wave equations which, in the semiclassical limit, can be described by ray trajectories
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Wave Chaos in Bounded Regions
Which Billiards Show Ray Chaos?

Consider a two-dimensional infinite square-well potential (i.e. a billiard) which shows chaos in the classical limit:

The statistical properties of eigenvalues and eigenfunctions of closed billiard systems are in excellent agreement with the predictions of Random Matrix Theory (RMT)

Can RMT work for open systems?

Can RMT include the effects of losses or “de-coherence” in real systems?

The Difficulty in Making Predictions in Wave Chaotic Systems...

We must resort to a statistical description. In our experiments we systematically move the perturber to generate many “realizations” of the system with extreme sensitivity to small perturbations.

\[ \frac{\lambda}{L} \sim 0.05 \]

Electromagnetic Wave Impedance

Abs[Z_{\text{cav}}] (Ω)

Abs[Z_{\text{cav}}]

Frequency [GHz]

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Microwave Cavity Analog of a 2DEG

Our Experiment: A clean, zero temperature, quantum dot with no Coulomb or correlation effects! Table-top experiment!

\[ \nabla^2 \Psi_n + \frac{2m}{\hbar^2} (E_n - V) \Psi_n = 0 \]

with \( \Psi_n = 0 \) at boundaries

\[ \nabla^2 E_{z,n} + k_n^2 E_{z,n} = 0 \]

with \( E_{z,n} = 0 \) at boundaries

Stöckmann + Stein, 1990
Doron+Smilansky+Frenkel, 1990
Sridhar, 1991
Richter, 1992

Microwave Scattering Experiment

The statistics of $z$ are “universal” and are described by Random Matrix Theory (RMT) 


$$Z = R + iX \iff iK$$

Wigner+Eisenbud (1947)

Reaction matrix

$$Z_{Rad} = R_{Rad} + iX_{Rad}$$

Measured with outgoing boundary condition

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$Z = Z_0 \frac{1 + S}{1 - S}$$

$$z = \frac{R}{R_{Rad}} + j \frac{X - X_{Rad}}{R_{Rad}}$$

Works for any number of ports/channels
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Quantum vs. Classical Transport in Quantum Dots

2-D Electron Gas

C. M. Marcus (1992)
electron mean free path >> system size

Ballistic Quantum Transport

\[ G = \frac{2e^2}{h} \sum_{n=1}^{N_1} \sum_{m=1}^{N_2} |S_{nm}|^2 \]

Landauer-Büttiker

Quantum interference → Fluctuations in \( G \sim e^2/h \)
“Universal Conductance Fluctuations”

An ensemble of quantum dots has a distribution of conductance values:

\[ P(G) = \frac{1/2}{\sqrt{G/(2e^2/h)}} \]  

\((N_1=N_2=1)\)

Incoherent Semi-Classical Transport

\[ \langle G \rangle = \frac{2e^2}{h} \frac{N_1N_2}{N_1 + N_2} \]

\(N_1=N_2=1\) for our experiment

\[ \langle G \rangle = \frac{2e^2}{h} \frac{1}{2} \]

\[ P(G) = \delta \left( G - \frac{2e^2}{h} \frac{1}{2} \right) \]
De-Phasing in Quantum Transport

Conductance measurements through 2-Dimensional quantum dots show behavior that is intermediate between:

- **Ballistic Quantum transport**
- **Incoherent Classical transport**

Why? “De-Phasing” of the electrons

One class of models: Add a “de-phasing lead” with $N_\phi$ modes with transparency $\Gamma_\phi$. Electrons that visit the lead are re-injected with random phase.

- $\gamma = 0$ Pure quantum transport
- $\gamma \to \infty$ Incoherent classical limit

**Equation:**

$$G = I_2 / (V_1 - V_2)$$

**Graph:**

1. $P(G)$ vs. $G/G_0$
2. $G_0 = 2e^2/h$
3. $\gamma = N_\phi \Gamma_\phi$

Büttiker (1986), Brouwer-Beenakker (1997)

We can test these predictions in detail:
The Microwave Cavity Mimics a 2-Dimensional Quantum Dot

Uniformly-distributed microwave losses are equivalent to quantum “de-phasing”

Brouwer+Beenakker (1997)

<table>
<thead>
<tr>
<th>Microwave Losses</th>
<th>Quantum De-Phasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss Parameter:</td>
<td>$\gamma = 0$ Pure quantum transport</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\gamma \to \infty$ Classical limit</td>
</tr>
</tbody>
</table>

$\alpha = \frac{\Delta \omega}{3\text{dB bandwidth of resonances}}$

Mean spacing between resonances

By comparing the Poynting theorem for a cavity with uniform losses to the continuity equation for probability density, one finds:

$4\pi \alpha \leftrightarrow \gamma$

Data

<table>
<thead>
<tr>
<th>Determination of $\gamma$</th>
<th>Fit to PDF(eigenvalues of $S^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma(\langle T \rangle)$</td>
<td>$\gamma \geq 10$ Theory</td>
</tr>
</tbody>
</table>

Brouwer+Beenakker PRB (1997)
Why does the Microwave Cavity ↔ Quantum Dot Analogy Work?

It is known that for electromagnetic systems:
Uniformly distributed losses are equivalent to a large number of “ports”, each with small transmission.

Lewenkopf, Müller, Doron (1992)
Schanze (2005)

Potential with uniform imaginary part

Physical Ports

Uniformly Lossy Cavity

Efetov (1995)
McCann+Lerner (1996)

Potential with uniform imaginary part

Potential with zero imaginary part

Parasitic Ports (Equivalent Loss Channels)

“Locally weak absorbing limit”

Zirnbauer (93)

Brouwer+Beenakker (1997)
Conductance Fluctuations of the Surrogate Quantum Dot

\[ \gamma_{\langle T \rangle} \approx 82.2 \]

Data (symbols)

\[ \gamma_{\langle T \rangle} \approx 35.1 \]

RMT predictions (solid lines) (valid only for \( \gamma \gg 1 \))

\[ \gamma_{\langle T \rangle} \approx 56.8 \]

High Loss / Dephasing

Ordinary Transmission

\[ \gamma_{\langle T \rangle} \approx 91.6 \]

Suppose the conductance fluctuations are described by a function \( P(G) \). The scaling prediction for \( P(G) \) is given by

\[ \log_{10}\left[ P(G) / G/(2e^2 / h) \right] = \log_{10}\left[ \frac{(1 + |x| - x)e^{-|x|}}{2} \right] \]

\[ \gamma_{\langle T \rangle} \approx 220.5 \]

\[ \gamma_{\langle T \rangle} \approx 6.91 \]

\[ \gamma_{\langle T \rangle} \approx 5.220 \]

\[ \gamma_{\langle T \rangle} \approx 272.1 \]

\( (x7) \)

\[ \gamma_{\langle T \rangle} \approx \frac{1}{\sqrt{2}} \left( 1 - |s_{11}|^2 - |s_{12}|^2 \right) \left( 1 - |s_{22}|^2 - |s_{21}|^2 \right) \]

\[ \gamma_{\langle T \rangle} \approx \frac{1}{\sqrt{2}} \left( 1 - |s_{11}|^2 - |s_{12}|^2 - |s_{21}|^2 - |s_{22}|^2 \right) \]

Correction for waves that visit the “parasitic channels”

Brouwer + Beenakker (1997)
Conclusions

The microwave analog experiment can provide clean, definitive tests of some theories of quantum chaotic scattering

Demonstrated the advantage of impedance (reaction matrix) in quantitative understanding of wave/quantum chaotic phenomena

Ongoing work on time-dependent scattering

Some Relevant Publications:


http://www.csr.umd.edu/anlage/AnlageQChaos.htm