Fundamentals of Microwave Superconductivity

Short Course Tutorial
Superconductors and Cryogenics in Microwave Subsystems
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Objective

To give a basic introduction to superconductivity, superconducting electrodynamics, and microwave measurements as background for the Short Course Tutorial “Superconductors and Cryogenics in Microwave Subsystems”
Outline

• Superconductivity
• Microwave Electrodynamics of Superconductors
• Experimental High Frequency Superconductivity
• Current Research Topics
• Further Reading
Superconductivity

- The Three Hallmarks of Superconductivity
- Superconductors in a Magnetic Field
- Where is Superconductivity Found?
- BCS Theory
- High-$T_c$ Superconductors
- Materials Issues for Microwave Applications
The Three Hallmarks of Superconductivity

Zero Resistance

Complete Diamagnetism

Macroscopic Quantum Effects

DC Resistance

Magnetic Induction

Flux \( \Phi \)

Flux quantization \( \Phi = n\Phi_0 \)

Josephson Effects

Temperature

Temperature

Temperature

Temperature
Zero Resistance

R = 0 only at $\omega = 0$ (DC)

R > 0 for $\omega > 0$

The Kamerlingh Onnes resistance measurement of mercury. At 4.15K the resistance suddenly dropped to zero.
Perfect Diamagnetism

Magnetic Fields and Superconductors are not generally compatible

The Meissner Effect

Spontaneous exclusion of magnetic flux

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) = 0 \]

\( \lambda(T) \)

magnetic penetration depth

\( \lambda(0) \)

\( \lambda(T_c) \)

\( \lambda \) is independent of frequency (\( \omega < 2\Delta \))

The Yamanashi MLX01 MagLev test vehicle achieved a speed of 343 mph (552 kph) on April 14, 1999
Macroscopic Quantum Effects

Superconductor is described by a single Macroscopic Quantum Wavefunction

\[ \Psi = |\Psi|e^{i\phi} \]

**Consequences:**

Magnetic flux is quantized in units of \( \Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} \text{Tm}^2 \)

R = 0 allows persistent currents

Current I flows to maintain \( \Phi = n \Phi_0 \) in loop

\( n = \) integer, \( h = \) Planck’s const., \( 2e = \) Cooper pair charge

Magnetic vortices have quantized flux

\( A \) vortex

\( B(x) \)

\( |\Psi(x)| \)

Type II

\( \xi << \lambda \)

\( 0 \xi \lambda \)

Line cut

Sachdev and Zhang, *Science*

vortex core

vortex lattice

screening currents

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Josephson Effects (Tunneling of Cooper Pairs)

\[ I = I_c \sin(\phi_1 - \phi_2) \]

DC Josephson Effect

\[ (\dot{\phi}_1 - \dot{\phi}_2) = \frac{e^* V_{DC}}{\hbar} \]

AC Josephson Effect

Quantum VCO:

\[ \frac{e^*}{h} = \frac{1}{\Phi_0} = 483.593420 \text{ MHz} \]

\[ \frac{\mu V}{\mu V} \]
Superconductors in a Magnetic Field

The Vortex State

The Vortex Lattice

\[ \vec{F}_L = \vec{J} \times \vec{\Phi}_0 \]

Lorentz Force

Moving vortices create a longitudinal voltage

Vortices also experience a viscous drag force:

\[ \vec{F}_{\text{Drag}} = -\eta \vec{v}_{\text{vortex}} \]
What are the Limits of Superconductivity?

Ginzburg-Landau free energy density

Temperature dependence

Currents

Applied magnetic field
BCS Theory of Superconductivity

Bardeen-Cooper-Schrieffer (BCS)

Cooper Pair

s-wave ($\ell = 0$) pairing

Spin singlet pair

First electron polarizes the lattice

Second electron is attracted to the concentration of positive charges left behind by the first electron

$$T_c \approx \frac{\Omega_{Debye}}{N} \, e^{-1/NV}$$

$\Omega_{Debye}$ is the characteristic phonon (lattice vibration) frequency

$N$ is the electronic density of states at the Fermi Energy

$V$ is the attractive electron-electron interaction

A many-electron quantum wavefunction $\Psi$ made up of Cooper pairs is constructed with these properties:

An energy $2\Delta(T)$ is required to break a Cooper pair into two quasiparticles (roughly speaking)

Cooper pair size: $\xi = v_F \cdot \frac{\hbar}{\Delta}$
Where do we find Superconductors?

Also:

Nb-Ti, Nb₃Sn, A₃C₆₀, NbN, MgB₂, Organic Salts ((TMTSF)₂X, (BEDT-TTF)₂X), Oxides (Cu-O, Bi-O, Ru-O,…), Heavy Fermion (UPt₃, CeCu₂Si₂,…), Electric Field-Effect Superconductivity (C₆₀, [CaCu₂O₃]₄, plastic), …

Most of these materials, and their compounds, display spin-singlet pairing
The High-Tc Cuprate Superconductors

Layered structure – quasi-two-dimensional
Anisotropic physical properties
Ceramic materials (brittle, poor ductility, etc.)
Oxygen content is critical for superconductivity

Spin singlet pairing
d-wave ($\ell = 2$) pairing

$\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$
Two of the most widely-used HTS materials in microwave applications

$\text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8$
HTS Materials Issues Affecting Microwave Applications

Most HTS materials made as epitaxial thin films for use in planar microwave devices

High-$T_c$ $\Rightarrow$ small Cooper Pair size ($\xi$ – correlation length)

$$\xi = \frac{v_F}{\Delta} \propto v_F \cdot \frac{1}{T_c}$$

$\xi \sim 1 – 2$ nm for HTS materials used in microwave applications

Superconducting pairing is easily disrupted by defects:
- grain boundaries
- cracks

Josephson weak links are created, leading to:
- nonlinear resistance and reactance
- intermodulation of two microwave tones
- harmonic generation
- power-dependence of insertion loss, resonant frequency, $Q$
Microwave Electrodynamics of Superconductors

• Why are Superconductors so Useful at Microwave Frequencies?

• The Two-Fluid Model

• London Equations

• BCS Electrodynamics

• Nonlinear Surface Impedance
Why are Superconductors so Useful at Microwave Frequencies?

**Low Losses:**
- Filters have low insertion loss ➔ Better S/N, can be made small
- NMR/MRI SC RF pickup coils ➔ x10 improvement in speed
- High Q ➔ Steep skirts, good out-of-band rejection

**Low Dispersion:**
- SC transmission lines can carry short pulses with little distortion
- RSFQ logic pulses – 1 ps long, ~2 mV in amplitude:

\[
\int V(t) dt = \Phi_0 = 2.07 \text{ mV} \cdot \text{ps}
\]

Superconducting Transmission Lines

\[ L = L_{\text{kin}} + L_{\text{geo}} \text{ is frequency independent} \]
Electrodynamics of Superconductors
In the Meissner State

\[ E \]
\[ 2\Delta \]
\[ 0 \]

Energy Gap

Quasiparticles (Normal Fluid)

Cooper Pairs (Super Fluid)

\[ \sigma = \sigma_n - i \sigma_2 \]

\[ \sigma_n = n_n e^2 \tau / m \]

\[ \sigma_2 = n_s e^2 / m \omega \]

\[ J = \sigma E \]

\[ n_n(T) \]

\[ n_s(T) \]

Current-carrying superconductor

\[ J = J_s + J_n \]

\[ \sigma_n \]

Normal Fluid channel

Superfluid channel

\[ L_s \]

\[ n_n = \text{number of QPs} \]

\[ n_s = \text{number of SC electrons} \]

\[ \tau = \text{QP momentum relaxation time} \]

\[ m = \text{carrier mass} \]

\[ \omega = \text{frequency} \]
Surface Impedance

\[ Z_s = R_s + iX_s = \frac{|\vec{E}_\parallel|}{\int |\vec{J}_\parallel(z)|dz} = \sqrt{\frac{i\omega \mu}{\sigma}} \]

Surface Resistance \( R_s \): Measure of Ohmic power dissipation

\[ P_{\text{Dissipated}} = \frac{1}{2} \text{Re} \left\{ \iiint_{\text{Volume}} \vec{J} \cdot \vec{E} \, dV \right\} = \frac{1}{2} R_s \iiint_{\text{Surface}} |\vec{H}|^2 \, dA \sim \frac{1}{2} I^2 R_s \]

Surface Reactance \( X_s \): Measure of stored energy per period

\[ W_{\text{Stored}} = \frac{1}{2} \iiint_{\text{Volume}} \left( \mu |\vec{H}|^2 + \text{Im}\{\vec{J} \cdot \vec{E}\} \right) \, dV = \frac{1}{2\omega} X_s \iiint_{\text{Surface}} |\vec{H}|^2 \, dA \sim \frac{1}{2} L I^2 \]

\[ X_s = \omega L_s = \omega \mu \lambda \]
Two-Fluid Surface Impedance

Because $R_s \sim \omega^2$:
The advantage of HTS over Cu diminishes with increasing frequency

$R_s$ crossover at $f \sim 100$ GHz at 77 K

$$R_s = \frac{1}{2} \omega^2 \mu_0 \lambda^3 \sigma_n$$

$$Z_s = R_s + iX_s$$

$$X_s = \mu_0 \omega \lambda$$
The London Equations

Newton’s 2nd Law for a charge carrier

\[ m \frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau} \]

\( \tau = \) momentum relaxation time
\( \vec{J}_s = n_s e \vec{v}_s \)

Superconductor: \( 1/\tau \to 0 \)

1st London Equation

\[ \frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m} \vec{E} = \frac{1}{\mu_0 \lambda_L^2} \vec{E} \]

1st London Eq. and yield:

\[ \frac{d}{dt} \left[ \nabla \times \vec{J}_s + \frac{n_s e^2}{m} \vec{B} \right] = 0 \]

London surmise

\[ \nabla \times \vec{J}_s + \frac{n_s e^2}{m} \vec{B} = 0 \]

2nd London Equation

These equations yield the Meissner screening

\[ \nabla^2 \vec{H} = \frac{1}{\lambda_L^2} \vec{H} \]

\( \vec{H} = \vec{H}_0 e^{\pm z/\lambda_L} \)

\( \lambda_L \approx 20 \text{ – } 200 \text{ nm} \)

\( \lambda_L \) is frequency independent \((\omega < 2\Delta)\)
The London Equations continued

<table>
<thead>
<tr>
<th>Normal metal</th>
<th>Superconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E ) is the source of ( J_n )</td>
<td>( \bar{J}_s = \frac{1}{\mu_0 \lambda_L^2} \bar{E} )</td>
</tr>
<tr>
<td>( \bar{J}_n = \sigma_n \bar{E} )</td>
<td>( \frac{d\bar{J}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} \bar{E} )</td>
</tr>
<tr>
<td>Lenz’s Law</td>
<td>( \mu_0 \lambda_L^2 \left( \nabla \times \bar{J}_s \right) = -\bar{B} )</td>
</tr>
<tr>
<td>( \frac{d}{dt} \left[ \nabla \times \bar{J}_n + \frac{1}{\mu_0 \lambda_L^2} \bar{B} \right] = 0 )</td>
<td>( \mathbf{B} ) is the source of ( J_s ), spontaneous flux exclusion</td>
</tr>
</tbody>
</table>

1\textsuperscript{st} London Equation \( \Rightarrow \) \( E \) is required to maintain an ac current in a SC
Cooper pair has finite inertia \( \Rightarrow \) QPs are accelerated and dissipation occurs
BCS Microwave Electrodynamics
Low Microwave Dissipation

Full energy gap $\rightarrow R_s$ can be made arbitrarily small

$$R_s \approx e^{-\Delta(0)/k_BT}$$

for $T < T_c/3$ in a fully-gapped SC

$$R_s = \frac{e^{\Delta_s}}{s}$$

$\Delta_s$ = energy gap

$R_{s,\text{residual}} \sim 10^{-9} \, \Omega$ at 1.5 GHz in Nb

HTS materials have nodes in the energy gap. This leads to power-law behavior of $\lambda(T)$ and $R_s(T)$ and residual losses

$$\lambda(T) = \lambda(0) + a \, T$$

$$R_s = R_{s,\text{residual}} + b \, T$$

$R_{s,\text{residual}} \sim 10^{-5} \, \Omega$ at 10 GHz in YBa$_2$Cu$_3$O$_{7-\delta}$

M. Hein, Wuppertal
Nonlinear Surface Impedance of Superconductors

The surface resistance and reactance values depend on the rf current level flowing in the superconductor.

Similar results for $X_s(B_s)$

Data from M. Hein, Wuppertal
Why are Superconductors so Nonlinear?

Granularity

Small $\xi \sim$ grain boundary thickness

Josephson weak links

Superconducting grains

JJs have a strongly nonlinear impedance

Intrinsic Nonlinear Meissner Effect

rf currents cause de-pairing – convert superfluid into normal fluid

$$\left( \frac{\lambda(0, T)}{\lambda(J, T)} \right)^2 = 1 - \left( \frac{J}{J_{NL}(T)} \right)^2$$

$J_{NL}(T)$ calculated by theory (Dahm+Scalapino)

Nonlinearities are generally strongest near $T_c$ and weaken at lower temperatures

Edge-Current Buildup

+ Vortex Entry and Flow

Heating

Scanning Laser Microscope image
YBCO strip at $T = 79$ K
$f = 5.285$ GHz, Laser Spot Size = $1 \mu m$

See poster 1EG08
How to Model Superconducting Nonlinearity?

(1) Taylor series expansion of nonlinear I-V curve (Z. Y. Shen)

\[
I(V) = I(0) + \left( \frac{dI}{dV} \right)_{V=0} \delta V + \frac{1}{2!} \left( \frac{d^2I}{dV^2} \right)_{V=0} \delta V^2 + \frac{1}{3!} \left( \frac{d^3I}{dV^3} \right)_{V=0} \delta V^3 + O(\delta V^4)
\]

\[= 0 \text{ if } I(-V) = -I(V)\]

1/R linear term

\[V = V_0 \sin(\omega t) \text{ input yields } \sim V_0^3 \sin(3\omega t) + \ldots \text{ output}\]

3rd order term dominates

(2) Nonlinear transmission line model (Dahm and Scalapino)

\[
\begin{align*}
\frac{\partial I}{\partial z} &= -C \frac{\partial V}{\partial t} \\
\frac{\partial V}{\partial z} &= -L \frac{\partial I}{\partial t} - RI
\end{align*}
\]

3rd harmonics and 3rd order IMD result

\[
\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2} + RC \frac{\partial I}{\partial t} + C \left( \frac{\partial L}{\partial t} \frac{\partial I}{\partial t} + \frac{\partial R}{\partial t} I \right)
\]

additional terms

L and R are nonlinear:

\[L = L_0 + \Delta L \left( \frac{I}{I_{NL}} \right)^2 \quad R = R_0 + \Delta R \left( \frac{I}{I_{NL}} \right)^2\]
Experimental Microwave Superconductivity

- Cavity Perturbation
- Measurements of Nonlinearity
- Topics of Current Interest
- Microwave Microscopy
Cavity Perturbation

Objective: determine $R_s$, $X_s$ (or $\sigma_1$, $\sigma_2$) from $f_0$ and $Q$ measurements of a resonant cavity containing the sample of interest

$\Delta f = f_0' - f_0 \propto \Delta$(Stored Energy)

$\Delta(1/2Q) \propto \Delta$(Dissipated Energy)

Cavity perturbation means $\Delta f \ll f_0$

$R_s = \frac{\Gamma}{Q}$

$\Delta X_s = \frac{2\Gamma}{\omega} \Delta \omega$

$\Gamma$ is the sample/cavity geometry factor
Intermodulation is a practical problem

Nonlinear (i.e., signal strength dependent) microwave response induces undesirable signals within the passband by intermodulation.

Measurement of Nonlinearities

Intermodulation harmonic generation

- Bandwidth of passband
- Intermodulation
- 3rd-order intercept Point (TOI)
- Harmonic generation

M. Hein, Wuppertal
Topics of Current Interest
In Microwave Superconductivity Research

Identifying and eliminating the microscopic sources of extrinsic nonlinearity
Increase device yield
Allows further miniaturization of devices
Will permit more transmit applications

Identify the additional Drude term now seen in $\sigma(\omega, T)$
under-doped cuprates show $\sigma_2 > 0$ above $T_c$
pseudo-gap electrodynamics

Nonlinear and Tunable Dielectrics
MgO substrates have a nonlinear dielectric loss at low temperatures
Ferroelectric and incipient ferroelectric materials as tunable
microwave dielectrics/capacitors
Microwave Microscopy of Superconductors

Use near-field optics techniques to obtain super-resolution images of:

1) Materials Properties: Nonlinear response
2) RF fields in operating devices

Scanning Laser Microscopy

Image $J_{rf}^2(x,y)$ in an operating superconducting microwave device

Image $J_{IMD}$
References and Further Reading

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