Stochastic Kron's model inspired from the Random Coupling Model

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***Abstract*— Through many applications, it was demonstrated that Kron's formalism gives to electromagnetic compatibility (EMC) challenges all the theoretical tools to analyse and to theorize complex systems electromagnetic coupling. Kron's formalism leads to describing electronic system with impedance matrices as fundamental objects. Recently the Random Coupling Method was proposed to describe the antenna impedance matrix in presence of wave chaotic cavities at high frequencies. In this paper, the integration of the Random Coupling Model inside the Kron's formalism is proposed, thus leading to a stochastic Kron's method.**

Keywords*— *Electromagnetic Computation; Chaotic Cavities; Statistical method;

I. Introduction

Electromagnetic Compatibility (EMC) is very important for Information Security [1-4]. Spurious compromising emanations [1-2] and high intensity parasitic fields are non-negligible threats for the security and the availability of critical systems. Immunity tests [3-4] can be performed for each electronic device that operates inside metallic enclosures as well as for the assembled electronic system as complex equipment. The electromagnetic waves radiated from a lot of intentional (e.g. wireless communication interface) and unintentional (e.g. electromagnetic noise) electronic devices will couple into the antenna of a receiving system, e.g. mobile communication emitters, or into the electronic device conductive structures. This coupling behavior can be seen as *front door* coupling into the antennas as well as *backdoor* coupling into the system cabling and the electronic itself. These parasitic coupling can disturb the system and may cause system errors or in the worst case the breakdown of the system as it was in-depth analyzed in [5].

In the design step of the electronic system, simulation tools are highly involved in order to prevent as much as possible parasitic coupling. To accurately simulate circuits influenced by the cavity eigenmodes along with the integrated electronics, studies rely generally on a full-wave solver involving circuit simulators. As these tools are known to be time-consuming, simple tools have been proposed to predict an estimation of the coupling between PCB's and parasitic fields [6]. Kron's formalism (KM) has shown its capability to provide EMC challenges [6-10] all the required theoretical tools for electromagnetic computation. The Kron's formalism in the meshes space is based on the use of Z-parameters matrix as a centered object in the theory of networks. Combined with the tensorial approach (TA), the KM forms a theoretical framework that can be efficiently employed to model complex EMC coupling problems.

Recently, a statistical model, based on impedance and/or admittance matrices (called the Random Coupling Model - RCM) has been derived to describe the coupling between ports and/or between apertures [11-14] within irregular resonating cavities. The hypothesis underlying the RCM is that the cavity supports a high density of ergodic eigenmodes, whose spectral statistics are chaotic and they can be described by semiclassics and random matrix theory.

It is argued that both formalisms, the KM and the RCM, can be combined in order to create an "all-impedance stochastic model", thus avoiding measurement of the environment scattering matrices. The KM impedance matrix of the source mesh and the KM impedance matrix of the load mesh can be coupled by the random impedance matrix of the RCM. Direct summation of KM networks for external measurements and environment and RCM impedance one representing equipment inside leads to the complete coupled system.

This is particularly appealing in those applications where the environment is partially unknown or inaccessible and requiring too much efforts for a deterministic modeling. If the free-space radiation impedance matrix of terminals and ports can be measured (e.g. inside an anechoic chamber), the coupling through the environment of subsystems connected to those ports and modeled through the KM can be predicted by Monte Carlo analysis of the RCM experiments.

As for a first validation purpose, the test case depicted in Figure 1 will be studied. The cavity have the following dimension 41.2 cm x 19.2 cm x 42.5 cm respectively along x, y and z axis. Two monopole antennas, represented as port 1 and port 2 (respectively as input and output ports during the measurements), are placed inside a metallic enclosure. Taking the lowest dimension of the cavity d_{low} = 19.2 cm, the maximal wavelength (i.e. the lowest usable frequency) taken in what follows λ (9.6 cm) >> d_{low}. This allows to comply with the overmoded hypothesis of the random coupling matrix. Nevertheless, as the instrumentation available allows for measuring a larger frequency band and since the Kron's method has no limitation the frequency band covered in this study is 4 GHz – 8 GHz. Note that in what follows, the simulation and measurement of the RCM will start at 4 GHz.

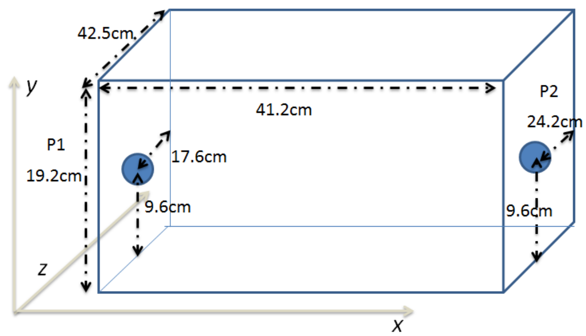

Fig. 1: Metallic electronic enclosure.

An impedance-matrix Kron's model of the two antennas inside the cavity will be given and simulation results will be compared to a set of measurements. The needs for a random coupling matrix will be illustrated. To overcome the challenge of characterizing and embedding the eigenmodes of the cavity inside Kron's model, it will be shown how to combine both the KM and RCM methods to build a stochastic Kron's model.

The paper is organized as follows: in Section II, Kron's principles are described and is applied to the analysis of the coupling between antennas placed inside a cavity. In Section III, the Random Coupling Model and its complementarity with Kron's simulation tools are presented.

II. Kron's Formalism

Kron's formalism is a theoretical tool that has been successful in describing the interactions within electromagnetic compatibility systems. Combined with the tensorial approach, the KM forms a theoretical framework that can be efficiently employed to model complex EMC coupling problems [7-8]. In this Section, the main lines of the KM method are recalled.

A. Kron's formalism

The Kron's formalism in the meshes space is based on the use of Z-parameters matrix as a center object in the theory of networks. This matrix leads to a Lagrange's equation of the problem. For this reason the technique is sometimes called a "modal method", compared to the "nodal" one which is used in SPICE for example and centered on the admittance expression of a circuit described in the nodes space.

Interestingly, the KM is in the mesh space and is based on the use of impedance operators. In contrast to SPICE models, where 2N nodal equations need to be solved, KM leads to a system of N Lagrange equations. KM works with three levels of space: the nodal, the edge and mesh ones. Nodal techniques as SPICE work only with the two first. Besides this, KM fundamental object is in impedance. This gives KM particular capacity: to translate directly the rotational operator for example and leads to Lagrange's equations of the network studied [9].

B. Simulation of the coupling between two antennas in a cavity

The problem can be written under the Kron's formalism in three steps: first step consists in locating dipoles in metallic walls. This leads to a network Z_d. Second step, it consists in making the direct summation of two similar dipoles Z_d and of two guided waves structure representing the cavity. Finally, it consists in adding the coupling between all previous elements.

The dipole being short ones, their near field interaction with a metallic plate stills the same until the plate is larger than a minimum dimension. This can be understood knowing that, if h is the height of the dipole, its impedance when short compared to the wavelength is a capacitor of value approximated given by:

$$C \approx 2\epsilon_0 r \ln(h). \quad (1)$$

This means that the first order influence on the dipole impedance is the plate zone of radius h. In our case, the distances all around the dipoles on the metallic walls of the cavity are larger than h. So we consider the impedance for a dipole located on the cavity walls defined by:

$$R_d + \frac{1}{C_d p} = Z_d. \quad (2)$$

The dipoles are seen as resonators. The generator has its own impedance R_0. The cavity structure is modeled through Branin's ones [10]. It consists of modelling lossy lines as a two port networks connected to load at both terminations. Thus, considering the case under study, Branin's models have been derived for modelling the cavity as short circuited waveguide. More details are accessible in [11]. Two polarizations are considered following the two directions of the plan parallel to the walls with the dipoles. Each polarization can be characterized by a mode number given by:

$$n_k = \frac{2L_k}{\lambda}, \quad (3)$$

where L_k is the width of the structure, a group speed [12] defined by:

$$vg_k = c\left(1 - \frac{n_k^2\pi^2c^2}{L_k^2\omega^2}\right), \quad (4)$$

and finally a characteristic impedance following Collin's meaning obtained with:

$$Z_c = \frac{2H_k}{L_k}\sqrt{\frac{\mu}{\epsilon}}. \quad (5)$$

These elements give all the information to model a guided waves structure using Branin's model [11]. The topology considered in this work is presented in Figure 2.

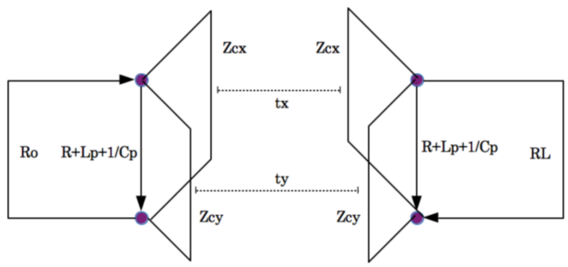

Fig. 2: Topology and associate graph describing the test case.

Next work is to add the mesh inductances and the coupling impedance to Z_b. These coupling impedances are magnetic one and Branin's coupling terms. Branin's impedance for the guide waves and k polarization are given by:

$$\begin{cases} \zeta_a = f_{ck}(\psi_b - Z_{ck}i_b)e^{-\frac{x}{vg_k}p}e^{-\alpha x} & (6.a) \\ \zeta_b = f_{ck}(\psi_a - Z_{ck}i_a)e^{-\frac{x}{vg_k}p}e^{-\alpha x}, & (6.b) \end{cases}$$

where ψ_u are the potential difference across the dipoles, f_{ck} cutoff functions defining the low cutoff frequency of the guided waves and ζ_u the voltage linked with the forward and backward waves transmitted by the guided waves.

Replacing ψ_u by their expressions depending on the dipole impedance and the current going across, we can define the coupling impedances. For example between ζ_a and i_b (equation 6.a), the coupling impedance is defined by:

$$Z_{ab} = Z_{ck}e^{-\frac{x}{vg_k}p}e^{-\alpha x}, \quad (7)$$

Once these coupling impedances defined between all the edges, they are added in the impedance matrix as extra-diagonal components. It stills to transform the impedance matrix into the mesh space and to solve the system.

Based on this computation we will be able to obtain the transfer function that we will be able to compare with measurements. In what follows, the transfer function is computed using the voltage at the RX monopole antenna (port 2) over the voltage at the TX one (port 1).

C. Transfer function measurement and simulation

The test case under study, depicted in Figure 1, is composed of two handmade monopole antennas covering a frequency band between 4 GHz and 6 GHz (the measured reflection coefficients of both antennas are below -15 dB). Both antenna are placed along x and have a length of 1.5 cm. The metallic enclosure is placed inside an anechoic chamber and the scattering parameters between port 1 and port 2 are obtained thanks to a vector network analyzer (VNA) from 1 GHz to 8 GHz with 8821 equally spaced points of measurement. Figure 3 shows the obtained simulated and measured transfer functions from 4 GHz to 8 GHz.

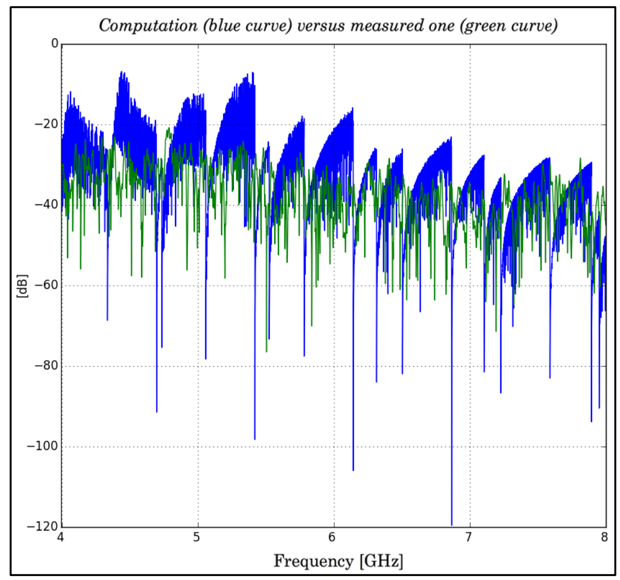

Fig. 3: Measured and simulated voltage magnitude transfer functions.

The modeled and the measured transfer functions are in good agreement between 4 GHz and 8 GHz, as the simple simulation software allows estimating the overall tendencies of the measured transfer function between the two monopoles. The global resonance density is retrieved as maximum levels. The mean error is about 9 dB. At this step, it can be observed that the cavity eigenmodes have not been added in the measurement as no mode stirrer has been used in contrary to the Kron's model which encloses the major modes involved through the antenna coupling functions.

Nevertheless, the cavity eigenmodes in Kron's model does not necessary strictly reflects the cavity response. In order to improve Kron's formalism, it is necessary to rely on a well-founded methodology. As a result, in the next section, it will be shown that it is possible to embed the cavity eigenmodes in Kron's formalism thanks to the Random Coupling Model in order to complete the free-space model, avoiding the characterization of the cavity.

III. Embedding the Eigenmodes of a cavity in Kron's formalism

A. Principle of the Random Coupling Model

The RCM works by first identifying a suitable set of voltages and currents that are linearly related and that can be used to describe the interaction of the fields within the cavity with signals to and from the outside world. The RCM then provides a model for the linear relation between port voltages and currents that mimics the behaviour of the fields in the enclosure. The model is based on the following approach. First one imagines representing the fields inside the enclosure in a complete basis of modes, and calculates the excitation of these modes due to coupling to the ports. One then writes a formal expression for the matrix impedance that involves the modes and their resonant frequencies. The mode topologies and resonant frequencies of a wave chaotic cavity are too complicated, and too sensitive to details, to calculate. So, these are replaced by representations that are based on random matrix theory [13-14] and the assumption that modes appear to be random superposition of plane waves. The result is a compact expression for a model of the cavity impedance matrix:

$$\underline{\underline{Z}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + [\underline{\underline{R}}^{rad\frac{1}{2}}\,\underline{\underline{\xi}}\,\underline{\underline{R}}^{rad\frac{1}{2}}] \tag{8}$$

where

$$\underline{\underline{Z}}^{rad} = \underline{\underline{R}}^{rad} + i\,\mathrm{Im}(\underline{\underline{Z}}^{rad}), \tag{9}$$

is in the simplest theory an N_p x N_p diagonal matrix (where N_p is the number of ports) whose elements are the complex radiation impedances of the ports, and we have adopted the notation that a double underline indicates a matrix quantity. Here the radiation impedance provides the linear relation between the voltages and currents at a port in the case in which waves are allowed to enter the enclosure through the port but not return, as if they were absorbed in the enclosure. $\underline{\underline{\xi}}$ is the only statistical matrix of $\underline{\underline{Z}}^{cav}$. Since the details of the EMC cavity environment can be too complicated to be estimated or simply unknown, the cavity eigenmodes are treated statistically. In particular, the matrix $\underline{\underline{\xi}}$ is an element of the Lorentzian ensemble [15] and can be defined for a lossless cavity as:

$$\underline{\underline{\xi}} = -\frac{i}{\pi}\sum_n \frac{\Phi_n\Phi_n^T}{K_0^2 - K_n^2 + i\alpha}, \tag{10}$$

Here $\underline{\Phi}_n$ is a vector of uncorrelated, zero mean, unit width Gaussian random variables, and K_n^2 are the eigenvalues of a matrix selected from the Gaussian Orthogonal Ensemble (GOE) [16], where the central eigenvalue is shifted to be close to $K_0^2 = \omega^2/c^2$ where ω is the frequency of excitation. The shift implies that $\underline{\underline{\xi}}$ has zero mean. The eigenvalues are scaled so that the average spacing between eigenvalues near the central one is ΔK^2, which is selected to match the mean spacing of resonances of the enclosure in the frequency range of interest. The effect of internal losses, or additional ports (beyond the N_p already considered) and the cavity dimension can be taking into account in the model thanks to the loss parameter $\alpha = K_k^2/Q.\Delta K^2$. It is through the matrix $\underline{\underline{\xi}}$ that the propagation of waves in the enclosure from one port to another and back is modelled. In the test scenario of Figure 1, and for arbitrary cavity losses, the elements of the random impedance matrix (8) take the explicit form:

$$Z_{11}^{cav} = i\,\mathrm{Im}(Z_{11}^{rad}) + R_{11}^{rad}\,\xi_{11}, \tag{11.a}$$

$$Z_{22}^{cav} = i\,\mathrm{Im}(Z_{22}^{rad}) + R_{22}^{rad}\,\xi_{22}, \tag{11.b}$$

$$Z_{12}^{cav} = (R_{11}^{rad})^{1/2}\,\xi_{12}\,(R_{22}^{rad})^{1/2}, \tag{11.c}$$

$$Z_{21}^{cav} = (R_{22}^{rad})^{1/2}\,\xi_{21}\,(R_{11}^{rad})^{1/2}. \tag{11.d}$$

B. Integration of the RCM model in the KM theory

The impedance matrix given by the RCM can be integrated into the KM in order to tackle the coupling between complex structures operating inside an arbitrary cavity. This can be carried out through a two-step procedure involving both the edge space and the mesh space.

First, since in RCM the ports are identified as independent edges, we include the input impedance of the two antennas as primitive objects in the edge impedance matrix, viz.,

$$Z_e = \begin{bmatrix} R_0 & 0 & 0 & 0 \\ 0 & Z_{11}^{rad} & 0 & 0 \\ 0 & 0 & Z_{22}^{rad} & 0 \\ 0 & 0 & 0 & Z_L \end{bmatrix}, \tag{12}$$

where the free-space radiation impedances are given by (2).

Given the connectivity matrix:

$$\mathrm{C} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \tag{13}$$

it is possible to represent the system in Figure 1 in mesh space.

The related impedance matrix is found by the bilinear transform:

$$Z_m = C^T Z_m C = \begin{bmatrix} R_0 + Z_{11}^{rad} & 0 \\ 0 & Z_L + Z_{22}^{rad} \end{bmatrix}, \qquad (14)$$

representing two uncoupled antennas radiating in free-space.

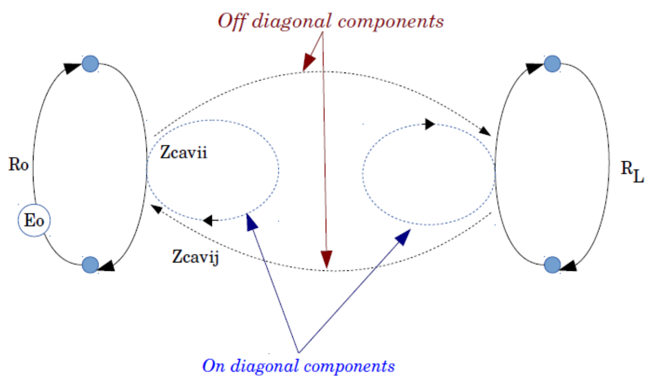

Fig. 4: Mesh representation of the two antennas radiating inside an arbitrary metallic cavity.

In the KM, the coupling is formalized through the interaction impedance Z_c, whose elements have the dimensions of impedance. By specializing the KM to the test scenario shown in Figure 1, and through the topology scheme given in Figure 4, we find:

$$Z_c = \begin{bmatrix} Z_{11}^{cav} & Z_{12}^{cav} \\ Z_{21}^{cav} & Z_{22}^{cav} \end{bmatrix}, \qquad (15)$$

where, because of reciprocity, the cross-impedances due to cavity wall reflections are given by (11.c) and (11.d). Finally, the electromotive forces (EMF) in the KM are found, given the source convector:

$$\mathrm{E} = [E_0 \quad 0], \qquad (16)$$

which can be obtained thanks to:

$$\mathrm{E} = (Z_m + Z_c).\mathrm{Q}, \qquad (17)$$

from which the mesh currents Q can be calculated. Note that the solution of (17) is subject to the Monte Carlo generation of the random cavity impedances in (14) through (15), whence (8) and (10) can be used to generate probability distributions of the mesh currents. Interestingly, since $<\underline{\underline{\xi}}> = \underline{\underline{0}}$, the ensemble average of (z) yields:

$$\mathrm{E} = < (Z_m + Z_c) >.<\mathrm{Q}> = Z_m.\mathrm{Q}\,, \qquad (18)$$

which retrieves the free-space result.

C. Introduction of the RCM in Kron's models

As for a first test case, we focused on the coupling of the two monopole antennas placed in a computer box as represented and described in Section I. Using equations 11.a to 11.b, the free-space model of the configuration under study has been completed to perform statistical analysis of the induced voltage on port 2 for a 10 V EMF applied on the port 1 monopole antenna at 4 GHz when both antennas are placed inside the cavity. As for preliminary results, 600 ergotic modes have been embedded in Kron's model. A 2000 random configurations (Monte Carlo analysis) run has been performed. A 1.0 loss factor has been defined as it is known to be a typical value for a 3D reverberation chamber in the few GHz range.

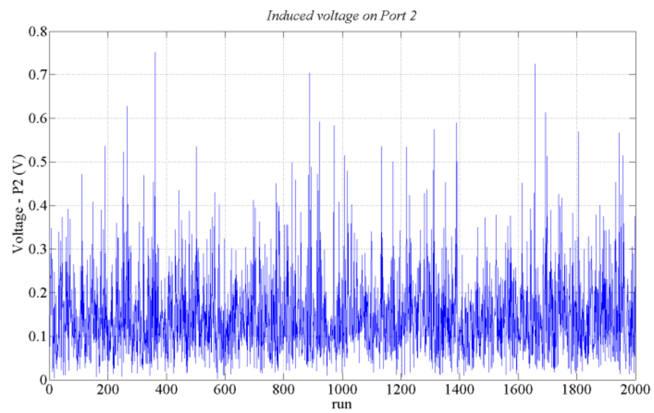

Fig. 5: Induced voltage magnitudes on port 2 for 2000 Monte-Carlo runs.

The voltage magnitude for each random configuration was obtained. Simulation results are depicted in Figure 5. The mean and standard deviation are 145.2 mV and 99.8 mV respectively.

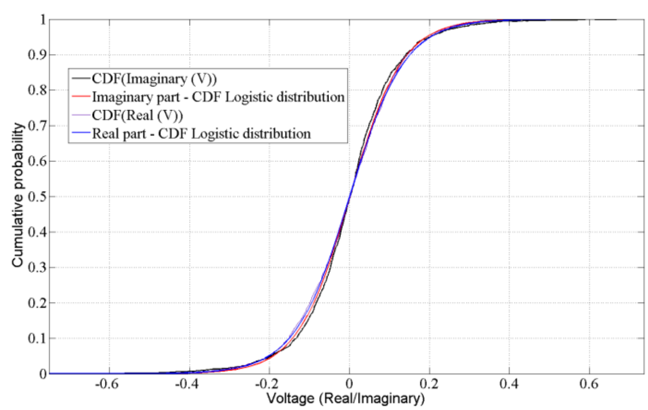

Fig. 6: Cumulative distribution function of the imagineray and real parts of the induced voltages on port 2 fitted with Logistic distributions.

The cumulative distribution functions (CDF) of the imaginary and real parts of the induced voltages on port 2 are depicted in Figure 6. Both distributions have been compared to known distribution. In this case, it has been confirmed thanks to the maximum log-likelihood function that the real and imaginary parts of the induced voltage are logistic distributions which are normal distribution with heavier tails.

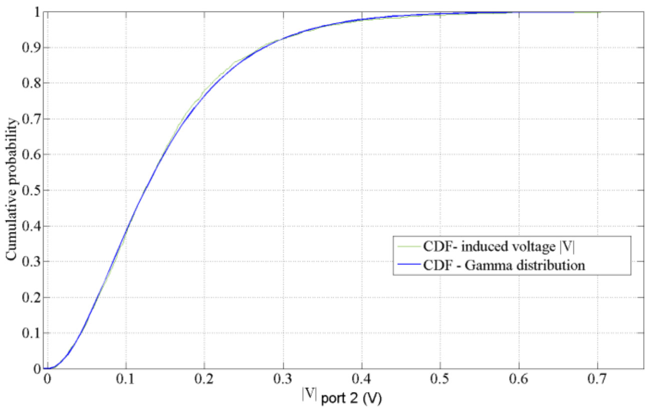

Fig. 7: Cumulative distribution function of induced voltage magnitudes on port 2 fitted with a Gamma distribution.

Based on the combination of the imaginary and real parts distributions, we expected to have a normal distribution with a larger shape parameter due to the outliers. This has been confirmed as the induced voltages magnitudes distribution is following a Gamma distribution as it is depicted in Figure 7.

IV. Conclusion

In this paper, the main lines of the Kron's formalism and the Branin's model have been recalled. Thanks to the Z-matrices enclosed in the Kron's formalism, it has been shown that for a reduced effort, the coupling between antennas placed inside metallic enclosure can be computed. The needs for embedding the eigenmodes of the cavity have been highlighted. To overcome the last challenge, it has been shown the Random Coupling Matrix can be enclosed in Kron's formalism in order to build a stochastic Kron's methods for electromagnetic computation. During the presentation, additional results will be provided in order to complete the theoretical aspects dealing with the RCM and the KM combination.

Future work will be dedicated to analyze and estimate experimentally the distribution of the induced voltages on a monopole antenna placed in a cavity. Moreover, the configuration presented in this study was composed of small dipoles, placed inside a cavity, for which the impedance is mostly reactive (i.e. capacitive). The work will be extended to the analysis for large dipole antennas.

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