Statistical Prediction and Measurement of Induced Voltages on Components within Complicated Enclosures: A Wave-Chaotic Approach

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Abstract—We consider induced voltages on electronic components housed inside complicated enclosures and subjected to high frequency radiation. The enclosure is assumed to be large compared to the wavelength in which case there is strong dependence of wave properties (eigenvalues, eigenfunctions, scattering, and impedance matrices, etc.) on small perturbations. The source(s) and sink(s) of radiation are treated as generalized ports and their coupling to the enclosure is quantified by an appropriate non-statistical radiation impedance matrix. The field fluctuations within the enclosure are described in a statistical sense using the hypothesis that these fluctuations conform to random matrix theory. The random matrix theory approach implies that the wave fluctuations have “universal” properties in the sense that the statistical description of these properties depends only upon the value of a single, experimentally-accessible, dimensionless loss-parameter. We formulate a statistical prediction algorithm for the induced voltages at specific points within complicated enclosures when subjected to short-wavelength electromagnetic energy from either external or internal sources. The algorithm is tested and verified by measurements on a computer box. The insights gained from this model suggest design guidelines for enclosures to make them more resistant to disruptive effects produced by a short-wavelength electromagnetic radiation.

Index Terms—Electromagnetic Compatibility, High-Power Microwave Effects, Overmoded Cavities, Radiation Impedance, Ray Chaos, Statistical Electromagnetism, Wave Scattering.

I. INTRODUCTION

Characterizing the nature of short-wavelength electromagnetic (EM) field quantities within large complicated enclosures connected to multiple ports (avenues of ingress or egress of EM energy) poses a unique challenge in the field of electromagnetic compatibility and microwave engineering. This problem manifests itself in many situations such as wireless-signal penetration into rooms or buildings [1], spurious electromagnetic emissions from personal electronic devices inside aircraft fuselages [2, 3, 4], or the upset of sensitive electronic systems due to intentional electromagnetic interference threats [5, 6]. The abundance of such situations has motivated a significant research effort in attempting to identify, quantify and eventually predict the nature of short-wavelength EM field quantities within large complicated enclosures. Currently, this effort can be broadly classified into two approaches, one that may be described as deterministic and another that may be described as statistical.

The deterministic approach makes use of sophisticated numerical analysis techniques to estimate the value of the EM field quantities at specific locations within the enclosure of interest, such as a room, aircraft fuselage or computer box. In this approach, the exact geometry of the ports which “drive” the enclosure as well as the geometry and location of objects or components within the enclosure are typically modeled using a CAD software tool. The numerical analysis software then proceeds to solve the relevant wave equations subject to appropriate boundary conditions for the desired EM field quantities within the enclosure. This deterministic approach has been facilitated by significant advances in the areas of computer hardware, computational electromagnetics, numerical analysis techniques, and data processing capabilities. In certain laboratory-type scenarios it is now possible to make deterministic predictions of short-wavelength...
EM field quantities inside electrically large enclosures, and these predictions agree reasonably well with experimental measurements [7].

In a specific application it is not always possible to identify or accurately model the geometry of all the driving ports and the location of all the objects within the enclosure. Moreover, when the wavelength of the electromagnetic radiation becomes much smaller than the size of the enclosure and/or the electromagnetic environment becomes very dense, such as within the avionics bay of an aircraft where several electrical cables lie in close proximity to one another, the computational time and resources required for a deterministic solution to the electromagnetic field quantities of interest can be quite prohibitive. In such large complicated enclosures, the nature of the bounded short-wavelength electromagnetic field quantities show strong fluctuations which are extremely sensitive to the detailed shape of the enclosure (which behaves as a cavity resonator), the orientation of internal objects (which act as scattering features), the frequency of the radiation, and the exact geometry of the ports. Minute changes in the shape of the enclosure, such as contractions or expansions due to ambient thermal fluctuations, mechanical vibrations or the reorientation of an internal component or cable can result in significantly different electromagnetic environments within the enclosure. Thus, even a deterministic solution to the electromagnetic response of the enclosure for one configuration will provide no information in predicting that of another nearly identical configuration. Hence a probabilistic approach is called for which treats the bounded electromagnetic field quantities as random variables and the nature of their fluctuations characterized by suitable probability density functions (PDFs). This approach has spawned the field of statistical electromagnetism [8].

Researchers in the field of statistical electromagnetism create probabilistic models for the PDFs of EM field fluctuations within large complicated enclosures making certain assumptions about the nature of the EM wave-scattering within the enclosure. For instance, it is assumed that these EM field fluctuations arise purely because of the nature of the short-wavelength plane waves randomly bouncing within the complicated large enclosure (a statistical condition called the “Random Plane Wave Hypothesis” [9]). The inclusion of these assumptions into the statistical models has led to several “universal” predictions for the PDF of EM fields at a point, the correlation function of fields at two points near each other, the Q of the enclosure, and the statistics for the scattering properties within the enclosure [8, 10, 11, 12, 13, 14, 15, 16]. By “universal”, we mean that the shapes and scales of the PDFs are not intimately dependent on the exact shape of the enclosure or the location of the scatters within the enclosure, but rather are parameterized by certain global quantities such as the total loss within the enclosure.

Researchers have also been successful in experimentally validating the existence of these universal fluctuations using mode-stirred chambers [17, 18] which inherently possess the necessary wave-scattering properties to produce such universal fluctuations [19], once it is experimentally ensured that the driving ports are perfectly coupled to the enclosure. By ‘perfect coupling’ we mean that a wave incident on the port from outside the enclosure is entirely transmitted into the enclosure experiencing no prompt reflection. Here we envision such an incident wave to arrive at the port through a connecting external transmission line, where the external transmission line may be an actual physical transmission line or a mathematical construct in which several incident modal waves are represented as arriving at the port via equivalent transmission lines. Thus in the perfect coupling case any reflected waves in these transmission lines originate from waves that have entered the enclosure, bounced within it, and returned to the port. Though the condition of “perfect coupling” may be achieved under special circumstances, this is generally not the case. Typically, the presence of mis-matched driving ports leads to system-specific artifacts in the measured EM field fluctuations at the ports, thereby leading to deviations from the predicted “universal” fluctuations. Thus when it comes to the problem of predicting short-wavelength EM field quantities at ports in large complicated enclosures, one should consider a statistical model which utilizes the universal aspects of the statistics of fields within the enclosure, but also accounts for the system-specific aspects introduced by the driving ports.

In this paper, we introduce such a model called the “Random Coupling Model” (RCM) as it applies to enclosures filled with reciprocal media (for enclosures filled with non-reciprocal media see Ref. [20, 21]). The foundation for a RCM treatment of statistical electromagnetic field properties is given in several previous publications [20, 21, 22, 23, 24]. Here, our objective is to develop the RCM into a quantitative prediction tool for the practical problem of calculating and mitigating the effects of electromagnetic interference within complicated metallic enclosures. In particular, we will provide an algorithm to predict the PDF (Probability Distribution Function) of the voltages induced at specific components within an enclosure. We will experimentally validate and illustrate our approach using the example of a computer-box irradiated by continuous-wave (CW), short-wavelength electromagnetic energy. The primary requirement for the validity of our approach is that the typical length-scale L of the enclosure be sufficiently large compared to the wavelength of the radiation \( \lambda_R \) (empirically, we typically obtain reasonable agreement with \( L/\lambda_R \geq 3 \)).

This paper is divided into the following sections. In Sec. II, we briefly discuss the underlying principles of the Random Coupling Model and present its salient features. Section III presents the general approach to calculating induced voltages in a system where the port has an arbitrary impedance. Section IV gives a brief overview of ways to determine the radiation impedance or admittance of an arbitrary port. In Section V, we present our experimental setup for the problem of EM coupling inside a computer-box. Section VI then reports the experimental validation of the RCM induced voltage algorithm for this computer-box setup. Based on the insights gained Section VII then presents design guidelines for enclosures that make them more resistant to interference from an internal or external short-wavelength electromagnetic source. Section
VIII then discusses the strengths, limitations and caveats associated with using the RCM on realistic enclosures such as computer-boxes, rooms or aircraft fuselages. Section IX concludes with a summary of the main points discussed in this paper. In Appendix A we outline our algorithm to generate the universal impedance fluctuations for a given cavity or enclosure using Random Matrix Theory.

II. THE “RANDOM COUPLING MODEL”

The Random Coupling Model (RCM) is formulated to address the problem of short-wavelength electromagnetic coupling into large, complicated metallic enclosures through multiple ports. This problem falls within a larger class of similar problems previously encountered in the fields of acoustics, quantum mesoscopic transport and nuclear physics [25]. All these systems have short-wavelength waves (electromagnetic, acoustic or quantum mechanical) which are trapped within an irregularly-shaped enclosure or potential well, in the case where the characteristic length of the system is substantially larger than the wavelength. In this limit, the waves within an enclosure can be approximated as rays which undergo specular reflections off the walls of the enclosure, much like the trajectory of a Newtonian point-particle elastically bouncing inside a similar-shaped enclosure. The dynamics of the rays within the enclosure depend on the shape of the enclosing boundaries, and an enclosure is said to be “ray-chaotic” if two typical rays launched with very slightly different initial conditions (slightly different initial location, or slightly different angular orientations), yield trajectories whose separation grows exponentially with distance along the ray during the time when this separation is small compared to the system size [26].

Ray chaos is common. Even very simple-shaped enclosures or cavities can produce chaotic ray dynamics [27, 28, 25]. The inherent complexities associated with the boundary shape of practical enclosures can easily create chaotic dynamics for the rays within the enclosure. A manifestation of wave chaos is the “soda can effect” in which the movement of an object on the scale of the wavelength inside an electromagnetic enclosure can dramatically alter the field strengths at remote locations within the enclosure.

A remarkable aspect of ray-chaotic systems is that despite their apparent complexity, they all possess certain universal statistical properties in their wave-scattering fluctuation characteristics [25, 29], and these statistical fluctuations are observed to be well-described by the statistical properties of ensembles of large random matrices [30, 31]. Thus, it is possible to simulate the statistical wave-scattering behavior of ray-chaotic systems by using Random Matrix Monte Carlo techniques.

For a ray-chaotic enclosure coupled to \( N_p \) ports, the Random Coupling Model characterizes the fluctuations in the impedance and scattering matrices. The scattering matrix \( S \) models the scattering region of interest in terms of a \( N_p \times N_p \) complex-valued matrix. Specifically, it expresses the amplitudes of the \( N_p \) outgoing scattered waves (\( \vec{b} \)) in terms of the \( N_p \) incoming waves (\( \vec{a} \)) at the location of each port (i.e., \( \vec{b} = S \vec{a} \)). The impedance matrix \( Z \) relates the complex voltages (\( V \)) at the \( N_p \) driving ports to the complex currents (\( I \)) at the \( N_p \) ports (i.e. \( V = ZI \)). The matrices \( S \) and \( Z \) are strongly fluctuating functions of frequency, and are related through the bilinear transformation,

\[
S = \left[ Z_o \right]^{1/2} \left( Z + Z_o \right)^{-1} \left( Z - Z_o \right)^{-1} \left[ Z_o \right]^{1/2} \tag{1}
\]

where \( Z_o \) is a \( N_p \times N_p \) real, diagonal matrix whose elements are the characteristic impedances of the transmission line input channels at the \( N_p \) driving ports.

It is convenient to think of modeling the statistical properties of the measured \( N_p \times N_p \) impedance matrix (\( Z_{\text{cav}} \)) of the ray-chaotic enclosure as consisting of (see Fig. 1)- a “core” universal, detail-independent fluctuating part that we call the “normalized impedance matrix (\( \xi Z \) )”; and an “outer shell” imposed by the system-specific coupling details of the ports that is quantified by a radiation impedance matrix (\( Z_{\text{rad}} \)) [21]. In terms of these quantities Zheng et al. [21] show that it is appropriate to model the impedance matrix (\( Z_{\text{cav}} \)) as:

\[
Z_{\text{cav}} = j \text{Im}[Z_{\text{rad}}] + [\text{Re}[Z_{\text{rad}}]]^{1/2} \cdot \xi Z \cdot [\text{Re}[Z_{\text{rad}}]]^{1/2}.
\]

Fig. 1: Schematic showing a convenient way to visualize the random coupling model. The statistical properties of the measured impedance of wave-chaotic enclosures coupled to multiple ports consists of a detail-independent, universally fluctuating normalized impedance (\( \xi Z \)), and the system-specific non-ideal coupling details of the measurement ports. This non-ideal port coupling is accurately quantified by the experimentally-accessible and non-statistical radiation impedance of the ports.

The radiation impedance matrix (\( Z_{\text{rad}} \)) is an experimentally-accessible, \( N_p \times N_p \) complex-valued matrix whose elements are non-statistical, smoothly-varying frequency-dependent quantities. The diagonal elements of
\( Z_{\text{rad}} \) quantify the detail-specific aspects of the coupling between the ports and the enclosure for arbitrary port geometries. The off-diagonal elements represent the cross-talk between the different ports [20]. The radiation impedance matrix \( Z_{\text{rad}} \) can be visualized in the following way: if the enclosure whose impedance is represented by Eq. (1) is driven by the same ports (having the same coupling geometry) as before, but has the distant side-walls of the enclosure and the internal scattering features moved out to infinity (or coated with a material that perfectly absorbs the incident waves), then the ports behave as a collection of free-space radiators. The boundary conditions corresponding to outgoing waves introduces a complex impedance matrix \( (Z_{\text{rad}} = R_{\text{rad}} + j \text{Im}[Z_{\text{rad}}]) \) at the plane of measurement of the driving-ports. This is illustrated schematically in Fig. (2) for an arbitrary enclosure coupled to two ports.

### a) Enclosure Case

![Schematic representation of a complicated enclosure coupled to two ports. The measured impedance of the enclosure \( (Z_{\text{cav}}) \) is a wildly fluctuating quantity on account of the waves originating from the two ports and experiencing multiple reflections before returning back to the ports. We refer to this condition as the “Enclosure Case”.](figure)

\[ Z_{\text{cav}} = R_{\text{cav}} + j \text{Im}[Z_{\text{cav}}] \quad Z_{\text{rad}} = R_{\text{rad}} + j \text{Im}[Z_{\text{rad}}] \]

### b) Radiation Case

![Schematic representation of the complicated enclosure in (a) which is coupled to two ports that retain their coupling geometry, but have the distant sidewalls coated with a perfectly absorbing boundary. The measured impedance of this enclosure \( (Z_{\text{rad}}) \) is now a smoothly varying function of frequency and arises on account of the waves originating from the two ports and radiating outwards without experiencing any internal reflections back to the ports. We refer to this condition as the “Radiation Case”.](figure)

The universally fluctuating quantity \( (Z_{\text{rad}}) \) corresponds to the impedance matrix of an enclosure which is “perfectly coupled” (as defined in Sec. 1) to its driving ports. Mathematically, perfect coupling refers to the situation when \( Z_{\text{rad}} = Z_{\alpha} \), where \( Z_{\alpha} \) is the diagonal matrix of characteristic impedances of transmission lines that connect to the ports. If \( Z_{\text{rad}} \) is known, Eq. (1) can be solved to determine the universal normalized impedance \( (\tilde{Z}) \) in terms of measured impedance matrices \( Z_{\text{cav}} \) and \( Z_{\text{rad}} \):

\[ \tilde{Z} = \left[R_{\text{rad}} \right]^{-1/2} \cdot \left[Z_{\text{cav}} - j \text{Im}[Z_{\text{rad}}] \right] \cdot \left[R_{\text{rad}} \right]^{-1/2} \quad (2) \]

The normalized impedance matrix \( (\tilde{Z}) \) corresponds to a normalized scattering matrix \( \tilde{S} = (\tilde{Z}^{-1}) \cdot (\tilde{Z} + 1)^{-1} \), and a normalized admittance matrix \( \tilde{Y} = \tilde{Z}^{-1} \). Here, 1 is the \( N_x N_y \) identity matrix, and the superscript “\(^{-1}\)” indicates a matrix-inversion operation.

According to the RCM, the only parameter on which the statistical properties of \( \tilde{Z} \), \( \tilde{S} \) or \( \tilde{Y} \) depends is the dimensionless enclosure loss-parameter \( (\alpha) \). For a large three-dimensional vacuum-filled electromagnetic enclosure, \( \alpha = k V / (2 \pi Q) \), where, \( k = 2\pi f / c \) is the wavenumber for the wave of frequency \( f \), and \( V \) represents the electromagnetic volume of the cavity. The quantity \( Q \) represents the typical loaded quality-factor of the enclosure and is defined as the ratio of the electromagnetic energy stored to the power leaving the unexcited cavity (due to ohmic and dielectric losses, as well as power leaving through the ports) per cycle. The loss-parameter \( \alpha \) can range from 0 (for a lossless enclosure) to \( \infty \). The eigenvalues of \( \tilde{Z} \) are correlated and have a joint probability distribution function (PDF) that can be calculated from random matrix theory (see Appendix A). From such a distribution function one can obtain marginal probability distributions for the real and imaginary parts of a single eigenvalue; we denote these marginal PDF’s by \( P_r(\lambda_{\tilde{Z}}) \) and \( P_i(\lambda_{\tilde{Z}}) \).

For an enclosure which is filled with a reciprocal medium having a real, symmetric permittivity or permeability tensor, Fig. 3 shows the evolution of these marginal PDFs with increasing values of \( \alpha \) [21], as calculated from random matrix theory. In the lossless case, \( Q = \infty \) \( (\alpha = 0) \), and the eigenvalues of \( \tilde{Z} \) are purely imaginary and \( P_i(\lambda_{\tilde{Z}}) \) is Lorentzian distributed with zero mean and unit full-width-at-half-maximum. As losses increase \( (\alpha > 0) \), \( \tilde{Z} \) develops a non-zero real part, and \( P_r(\lambda_{\tilde{Z}}) \) evolves from being peaked near zero for low \( \alpha \) toward being a Gaussian distribution that peaks at \( \text{Re}(\lambda_{\tilde{Z}}) = 1 \) for large \( \alpha \) (Fig. 3(a)). At the same time, \( P_i(\lambda_{\tilde{Z}}) \) loses its long tails (associated with the Lorentzian distribution at \( \alpha = 0 \)) and begins to sharpen up, developing a Gaussian appearance at large \( \alpha \) (Fig. 3(b)). For
all values of \( \alpha \), the mean value of \( \text{Re}(\lambda_{\xi Z}) \) (denoted \( \langle \text{Re}(\lambda_{\xi Z}) \rangle \)) is equal to 1, while the mean value of the \( \text{Im}(\lambda_{\xi Z}) \) (denoted \( \langle \text{Im}(\lambda_{\xi Z}) \rangle \)) is equal to 0. Approximate analytic expression for \( P_R(\lambda_{\xi Z}) \) and \( P_I(\lambda_{\xi Z}) \) for all values of \( \alpha \) are given in [32]. For the results discussed in this paper, we will employ a random matrix Monte Carlo approach [33, Appendix A] to generate the theoretical RCM predictions for \( P_R(\lambda_{\xi Z}) \) and \( P_I(\lambda_{\xi Z}) \).

![Fig. 3: Random Coupling Model predictions for the marginal probability density functions of the (a) real \( \text{Re}(\lambda_{\xi Z}) \) and (b) imaginary \( \text{Im}(\lambda_{\xi Z}) \) parts of the eigenvalues of \( \xi_{\Sigma} \) as a function of increasing loss-parameter \( \alpha \), for a wave-chaotic enclosure filled with a medium having real, symmetric permittivity or permeability tensors.](image)

For \( \alpha \gg 1 \), [21] predicts that the variance of \( \text{Re}(\lambda_{\xi Z}) \) (denoted \( \sigma_{rz}^2 \)) is approximately equal to the variance of the marginal PDF of \( \text{Im}(\lambda_{\xi Z}) \) (denoted \( \sigma_{iz}^2 \)). The magnitude of these variances depend only upon the value of the cavity loss-parameter \( \alpha \) and the presence of only reciprocal media, i.e.,

\[
\sigma_{rz}^2 = \sigma_{iz}^2 \approx \frac{1}{\pi \alpha} \quad \text{for} \quad \alpha \gg 1.
\]

Equation (3) has been experimentally validated in [22, 33] for \( \alpha > 5 \), and will be assumed to hold true for the results presented in this paper.

The existence of the universal fluctuations in the normalized impedance, scattering and admittance properties of one-port and two-port, quasi-two-dimensional ray-chaotic enclosures has been experimentally validated [22, 23, 24], and shown to be in agreement with predictions of the Random Coupling Model. A similar statistical EM wave-model describing only the fluctuations in the impedance of large complicated one-port enclosures has been presented in [34, 35]. This model uses the “terminal impedance” (similar in principle to the radiation impedance) of the port to quantify the non-ideal port coupling over narrow frequency bands, and derives an expression similar to Eq.(1). The RCM, however, incorporates all the predictions of [34, 35] and goes further to include complicated enclosures coupled to multiple-ports over arbitrarily large frequency ranges. This aspect of RCM makes it possible to consider the induced voltage and current statistics inside complicated enclosures driven by multiple ports.

### III. FORMULATING THE RCM INDUCED VOLTAGE

In the case of a computer-box, high frequency electromagnetic radiation can couple into the system through several independent channels (connectors, cooling vents, exposed wires, etc.). Also, internal components such as printed circuit board (PCB) tracks and integrated circuits can emit their own EM radiation. All these channels set up complicated standing wave patterns within the metallic enclosure of the computer box. The RCM can be applied to this problem through two levels of abstraction. At the first level of abstraction, we treat all relevant sources and sinks of radiation as “generalized ports.” By “relevant” we mean those discrete components or features that are actively adding (or taking away) energy to (from) the system, which are classified as either source (target) ports respectively. For instance, in the scenario of an external plane wave incident on a computer-box, a relevant source-port may be the cooling-vent (aperture) which allows the plane-wave energy to couple into the box, while a target-port might be the PCB bus-track (microstrip antenna) which carries the coupled energy to a sensitive IC chip. The problem of determining the radiation impedances of the relevant ports can be challenging. In the next section we briefly summarize methods to determine the radiation impedance of relevant ports based on their physical-geometries.

Unlike a regular microwave cavity-resonator where the ports tend to be at the interface between the external environment and the cavity-resonator; here a generalized port can lie entirely within the confines of the cavity-resonator. The presence of other components (memory cards, wire cables, etc.) within the computer-box will be accounted for in the model through the scattering of the waves they produce within the enclosure, as well as modifications to the enclosure volume and quality factor \( (Q) \). Once the relevant generalized ports have been identified, the second level of abstraction treats the computer box as a wave-chaotic enclosure with \( N_p \) ports, where \( N_p \) corresponds to the sum of all generalized ports.

In what follows, we explain the framework of the RCM induced voltage algorithm taking the example of a computer-box with two relevant generalized ports \( (N_p = 2) \) denoted as “port 1”(source) and “port 2”(target). Generalization to an arbitrary number of ports is straightforward. The algorithm essentially requires only three pieces of information in order to make accurate statistical predictions for the induced voltages at the target-port for a specified continuous-wave excitation at port 1 (see Fig. 4). These three quantities are,

1. The loss-parameter value (\( \alpha \)) for the enclosure.
2. The 2x2 radiation-impedance matrix of the source and target ports within the enclosure at the
frequencies of interest \( Z^{\text{rad}} (f) \). Since \( Z^{\text{rad}} (f) \) is a non-statistical quantity, it can either be directly measured (as described in Sec. V), determined analytically, or determined numerically using conventional EM-solver software. In this paper, we resort to experimentally measuring the radiation impedance of the relevant ports.

3. The frequency \( (f) \)-dependence of the radiated power at the source port \( \tilde{P}_1(f) \).

\[ \frac{Z^{\text{rad}}}{Z} \] is the radiation impedance matrix of source and target ports \( Z^{\text{rad}} \), and the frequency-dependence of the radiated-power at port-1 \( \tilde{P}_1(f) \). The loss-parameter \( \alpha \) is dependent on the frequency \( f \), enclosure volume \( V \) and typical enclosure quality-factor \( Q \) at frequency \( f \).

For a three-dimensional wave-chaotic enclosure, the loss parameter is \( \alpha = k^2 V / (2 \pi^2 Q) \). The value of \( \alpha \) in turn dictates the shapes and scales of the normalized impedance matrix \( \xi_Z \) element PDFs (Fig. 3), and can be numerically generated using random matrix Monte Carlo simulations [33, Appendix A]. The numerically derived ensemble of normalized impedance \( \xi_Z \) matrices can then be combined with the measured/calculated radiation-impedance matrix \( Z^{\text{rad}} \) and Eq. (1), to yield a numerical estimate of the PDF of the system-specific cavity-impedance matrix

\[
\left( \begin{array}{cc}
Z^{\text{cav}}_{11} & Z^{\text{cav}}_{12} \\
Z^{\text{cav}}_{21} & Z^{\text{cav}}_{22}
\end{array} \right) \]

Finally, using the formalism of the ABCD-transmission parameters for a two-port microwave network [36], it is possible to determine the PDF of the magnitude of the induced voltage at a target-port with impedance \( Z_L \) \( (P_1[|V_2|]) \) for the specified frequency-dependence of the radiated-power \( \tilde{P}_1(f) \) at the source-port through,

\[
|V_2| = \sqrt{\frac{2\tilde{P}(f) |Z_p|^2 |Z^{\text{cav}}_{11}|^2}{\text{Re}[Z^{\text{cav}}_{11}]}}.
\]

Where,

\[
Z_p = \frac{Z^{\text{cav}}_{12}Z_L/Z_{eq}}{Z^{\text{cav}}_{22} + Z_L} \quad \text{and} \quad Z_{eq} = \frac{Z^{\text{cav}}_{12}Z^{\text{cav}}_{11}}{Z^{\text{cav}}_{22} + Z_L}.
\]

In the limit where the load impedance approaches infinity (open-circuit on port 2), the induced voltage simplifies to

\[
|V_2| = \sqrt{\frac{2\tilde{P}(f) |Z^{\text{cav}}_{21}|^2}{\text{Re}[Z^{\text{cav}}_{11}]}}.
\]

For this last expression we assume port-2 to be open-circuited so that the only voltage present on port-2 is induced due to the excitation from port-1 and not influenced by any sources (or load impedances) which may be connected to port-2. We will use this limit in the calculated induced voltage distributions presented below.

IV. ESTIMATING THE RADIATION IMPEDANCE AND/PR ADMITTANCE OF TYPICAL COUPLING STRUCTURES

In this section we briefly review the ways in which the impedance (or admittance) matrix is defined for a port, and then discuss how its values are determined analytically. We then consider alternative methods to estimate the radiation impedance/admittance matrix of complicated ports using computational electromagnetic codes, as well as through direct experiments. The appropriate method to be adopted is in most cases dependent on the complexity of the radiating structure and other logistical factors such as experimental set-up time.

Under the analytic approach, we generally identify three situations of interest, which we label the terminal case, the closed aperture case, and the open aperture case, as shown in Fig. 4.5. The precise definition of the impedance matrix will vary in these cases, as will the method of calculation of the matrix. However, all three of these cases can still be treated within the Random Coupling Model.
The analytic approach for estimating the radiation impedance of simple radiating structures has been used to estimate the radiation impedance of monopoles [38], short and long dipole radiators [39, 34, 35], horn antennas [40], and microstrip antennas [41]. However, when the internal geometry of the enclosure surrounding the port becomes more complicated such as due to the presence of metal side walls or dielectric features in the near-field of the radiating ports, the analytic approximations can become too cumbersome to evaluate. In such cases, finite-element numerical-electromagnetic approaches based on time-domain or frequency-domain solvers can be utilized [42]. This numerical approach has proved to be successful for modeling aperture coupling within large enclosures such as automobiles [43] and computer boxes [44, 45]. Commercially available EM solver codes now have the ability to model very complex EM coupling scenarios, such as estimating the coupling between the pins of an IC chip [46, 47]. By using a combination of these numerical solver codes and the random coupling model, it is possible to come up with statistical descriptions for induced voltages on the pins of ICs for given excitation stimulus on an aperture-type cooling vent. Complications can arise if the near-by lead-pins are in different time-varying states i.e., logic-high state, logic-low state, or transitioning from one logic state to another. This aspect of the problem is presently beyond the capabilities of the frequency-domain RCM induced voltage algorithm and deserves further investigation.

IV (A) Terminal Case

The terminal case (Fig. 5 (a)) applies to the situation where a port is excited through a single mode transmission line, and the excitation of the port can be prescribed by a single variable: the voltage, or current, or amplitude of the incident wave on the transmission line. Our studies of the excitation of cavities by signals on cables, as described in Section VI, are examples of this case. In addition, a terminal or lead on an integrated circuit can be treated as an example of this case if one considers the input to the circuit as a lumped element and the conductors and dielectric material surrounding the integrated circuit as an antenna. In the terminal case, determination of the radiation impedance becomes equivalent to solving for the fields surrounding an antenna that is driven by a transmission line. It is thus important to account for the geometry and dielectric properties of the material surrounding, within several wavelengths, the terminal. Calculation of the port impedance can be quite complicated as it involves the self-consistent determination of the current in all conductors and polarization of all dielectrics near the port. A simple case is that of an antenna that is small compared with a wavelength. In this case the current distribution in the antenna is fixed. An example of this is that of a coaxial antenna in a two dimensional cavity [20, 21]. Further details about calculating the radiation impedance in the terminal case can be found in [37].

IV (B) Aperture Cases

The closed aperture case (Fig. 5(b)) applies to situations in which the cavity is excited through an aperture that is connected to a waveguide. In this case the port is characterized by an impedance (or scattering) matrix that has a dimension equal to the number of modes used to represent the fields in the aperture. The open aperture case (Fig. 5 (c)) applies when the aperture is illuminated by a plane wave incident with a wave vector $\mathbf{k}_{inc}$ and polarization of magnetic field $\mathbf{H}_{inc}$ that is perpendicular to $\mathbf{k}_{inc}$. These cases are treated in detail by [37].
can affect the repeatability and reliability of the experimentally measured radiation impedances.

As shown in Section VI of this paper, one can experimentally determine an approximate radiation impedance by taking an ensemble average of cavity impedances. This requires that the ensemble includes enough distinct realizations of the system to destroy the contributions of short orbits [49, 50, 51].

V. EXPERIMENTAL SETUP

We have performed experiments to validate the RCM induced-voltage algorithm as described in Sec. III. The three-dimensional enclosure under study is a typical computer-box of physical outer dimensions 38 cm x 21 cm x 23 cm (Fig. 6(a)), which contains all the internal electronics – motherboard, memory chips, network card, etc. (Fig. 6(b)). The floppy-drive, CDROM-drive and SMPS power-supply unit were removed to increase the internal volume of the enclosure and also to decrease the inherent enclosure-loss. The enclosure was excited by means of two ports, labeled Port 1 and Port 2 in Fig. 6(c), located on the top and bottom walls of the box. We consider the frequency range of 4 to 20 GHz. The free-space wavelength at 4 GHz corresponds to about 7.5 cm which is about three times smaller than the smallest enclosure dimension. The ports are sections of coaxial transmission lines which act as dipole radiators with the exposed inner-conductor of diameter 1.27 mm, extending 1.3 cm into the volume of the enclosure from the side walls.

\[
S_{cav} = \begin{bmatrix}
S_{11}^{cav} & S_{12}^{cav} \\
S_{21}^{cav} & S_{22}^{cav}
\end{bmatrix}
\]

matrix (using an Agilent E8364B Vector Network Analyzer. This is referred to as the “Enclosure Case”. To realize this large ensemble, a mode-stirrer is introduced into the volume of the enclosure. The mode-stirrer (shown in schematic in Fig. 6(c)) consists of a central metallic shaft (shown as the black line) of diameter 5 mm with two paddle-wheel type blades (gray-colored rectangles) measuring approximately 10 cm x 5 cm and placed 7 cm apart. The two blades are made of cardboard-paper coated with aluminum foil (Fig. 6(d)) and are oriented perpendicular to each other on the shaft. Each orientation of the blades within the cavity results in a different internal field configuration. For each configuration, \( S_{cav} \) is measured as a function of frequency from 4 to 20 GHz in 16000 equally spaced steps. By rotating the shaft through twenty different positions, an ensemble of 320,000 computer-box enclosure scattering matrices is thus collected. From the \( S_{cav} \) measurements, it is inferred that the typical loaded-Q of the computer-box enclosure ranges from about 45 at 4GHz to about 250 at 20GHz.

The port radiation-impedance measurement involves simulating an outward radiation condition for the two driving ports, but retaining the coupling structure as in the “Enclosure Case”. To achieve this condition, the mode stirrer is removed and all internal electronics are coated with a microwave foam absorber (Eccorsorb HR-25). A small circular region (about 8 cm in radius) around each driving port (not visible in photograph) is left uncoated to retain the near-field structure of the driving ports.

To make a statistical analysis of the electromagnetic response of the computer-box enclosure, the first step involves measuring a large ensemble of the full 2x2 enclosure scattering
The dashed enclosure case" (Fig. 8(a)). As can be seen in the figure, the two ports have rather different frequency-dependent coupling characteristics (indicated by the circles and triangles for port 1 and port 2 respectively). The dashed-black line, solid-gray line and solid-black lines represent the magnitude of the measured radiation-scattering elements $|S_{11}^{rad}|$, $|S_{22}^{rad}|$ and $|S_{21}^{rad}|$ respectively, which closely follow the general trend of the respective ensemble averaged scattering elements over the entire frequency range. This indicates that the radiation scattering matrix (or equivalently the radiation impedance) elements accurately quantify the non-ideal coupling between the ports and the computer-box enclosure at all frequencies. The slight oscillatory nature of the radiation-scattering matrix elements is attributed to imperfections in the absorber properties, allowing a small amount of wave energy to travel from a port to either itself or another port by following a path that reflects from the absorber [54, 55, 24, 49, 50, 51].

VI.(B): Characterizing the non-ideal port coupling through the measured radiation scattering matrix

In Figure 9, the average measured scattering matrix elements and the measured scattering matrix elements in the radiation case are shown as a function of frequency. The circles, triangles and pentagons represent the magnitude of the ensemble-averaged computer-box “Enclosure Case” $|\langle S_{ii}^{cav} \rangle |$, $|\langle S_{i2}^{cav} \rangle |$ and $|\langle S_{2i}^{cav} \rangle |$ elements respectively. The magnitude of the ensemble averaged scattering matrix elements is indicative of the degree of non-ideal coupling between the ports and the cavity [52,53]. A frequency range where the coupling between port-i and the cavity is good, results in small values of $|\langle S_{ii}^{cav} \rangle |^2$ ($i = 1, 2$). As can be seen in the figure, the two ports have rather different frequency-dependent coupling characteristics (indicated by the circles and triangles for port 1 and port 2 respectively). The dashed-black line, solid-gray line and solid-black lines represent the magnitude of the measured radiation-scattering elements $|S_{11}^{rad}|$, $|S_{22}^{rad}|$ and $|S_{21}^{rad}|$ respectively, which closely follow the general trend of the respective ensemble averaged scattering elements over the entire frequency range. This indicates that the radiation scattering matrix (or equivalently the radiation impedance) elements accurately quantify the non-ideal coupling between the ports and the computer-box enclosure at all frequencies. The slight oscillatory nature of the radiation-scattering matrix elements is attributed to imperfections in the absorber properties, allowing a small amount of wave energy to travel from a port to either itself or another port by following a path that reflects from the absorber [54, 55, 24, 49, 50, 51].

\[
S^{rad} = \begin{bmatrix} S_{11}^{rad} & S_{12}^{rad} \\ S_{21}^{rad} & S_{22}^{rad} \end{bmatrix},
\]
from 4 GHz to 20 GHz with the same 16000 frequency steps as in the “Enclosure Case”. This is a single realization, deterministic (non-statistical) measurement.

VI. EXPERIMENTAL RESULTS

The objective of the current section is to experimentally show the applicability of the “RCM Induced Voltage Algorithm” to address the practical problem of predicting induced voltage PDFs at specific target-ports within complicated enclosures, such as a computer-box. To achieve this objective, we first need to establish three aspects of the EM wave-scattering within the enclosure. First, we need to experimentally prove the existence of wave-chaotic scattering (Sec. VI.(A)). Second, the applicability of the port radiation impedance (scattering) matrix to account for the frequency-dependent coupling in a three-dimensional enclosure, where polarization of the waves and the effects of field variations associated with the presence of side-walls has to be established experimentally (Sec. VI.(B)). Third, the existence of universal fluctuations in the normalized impedance matrix for the computer-box enclosure has to be established and shown to be in agreement with corresponding predictions from RCM (Sec. VI.(C)). Finally, in Sec. VI.(D), we provide experimental results that validate the “RCM induced voltage algorithm” for the experimental setup discussed in Sec. V.

VI.(A): Establishing the existence of “Wave-chaos” inside the computer-box enclosure

In an enclosure, wave chaos manifests itself as extreme sensitivity of the internal field quantities to small changes in the wave frequency and in the enclosure’s internal configuration. For the computer-box enclosure this can be inferred by estimating the ratio of the maximum transmitted power to the minimum transmitted power at each frequency for the twenty different positions of the mode-stirrer. This power-ratio, denoted as $\Lambda = \max(|S_{21}^{cav}(f)|^2)/\min(|S_{21}^{cav}(f)|^2)$ (shown on a log-scale in Fig. 8), has a distribution which is fairly wide with a mean of 17.3 dB and a standard deviation of 6.2 dB (Fig. 8(a)). The dynamic range of $\Lambda$ is nearly 55 dB over the frequency range of 4 to 20 GHz (shown as the circles in Fig. 8(b)). This indicates that there are significantly large field fluctuations within the computer-box enclosure as the mode-stirrer is rotated, characteristic of wave-chaotic systems.
eigenvalues. Thus, if there are $X_{\text{num}}$ matrices in the ensemble, there are $2X_{\text{num}}$ eigenvalues in the measured ensemble, which are placed together into a list. We have observed that using both eigenvalues in the list, as opposed to randomly considering one of the two eigenvalues, does not alter the statistical results that follow. Histogram approximations to the Probability Density Functions of the real ($\Re(\lambda_{\xi Z})$) and imaginary ($\Im(\lambda_{\xi Z})$) parts of the eigenvalues of $\xi Z$ appearing on the list are shown in Fig. 10(a) and Fig. 10(b) respectively by the star symbols. The variance ($\sigma^2$) of experimental PDFs in Fig. 10(a) and Fig. 10(b) are nearly identical in magnitude, i.e., $\sigma_{\Re}^2 \approx \sigma_{\Im}^2 = 1.35 \times 10^{-3}$. From the variance of the PDFs of the real ($\Re(\lambda_{\xi Z})$) and imaginary ($\Im(\lambda_{\xi Z})$) parts of the eigenvalues of $\xi Z$, and by using Eq.(3), we estimate a value of the cavity loss-parameter ($\alpha$) for this data-set to be $\alpha \approx 236$.

Using the value of $\alpha = 236$, a random matrix Monte Carlo simulation [Appendix A] yields the black curves shown in Fig. 10(a) and Fig.10(b) for the real and imaginary parts of $\xi Z$ PDFs i.e., $P_R(\lambda_{\xi Z})$ and $P_I(\lambda_{\xi Z})$ respectively. Good agreement is observed between the experimentally derived PDFs and those generated numerically from Random Matrix Monte Carlo simulations. This agreement also extends to the other ~30 “data-sets” examined over the frequency range of 4 to 20 GHz, and supports the existence of universal fluctuations in $\xi Z$ for the computer-box enclosure.

![Fig. 9: Measured computer-box enclosure scattering matrix elements as a function of frequency. The circles, triangles and pentagons represent the magnitude of the ensemble-averaged computer-box cavity $|\langle S_{21}^{\text{cav}} \rangle|$. $|\langle S_{11}^{\text{cav}} \rangle|$ and $|\langle S_{21}^{\text{rad}} \rangle|$ elements respectively. The dashed-black line, solid gray line and solid black line represent the magnitude of the measured radiation-scattering elements $|S_{11}^{\text{rad}}|$, $|S_{21}^{\text{rad}}|$ and $|S_{21}^{\text{rad}}|$ respectively, which closely follow the general trend in the corresponding ensemble-averaged enclosure scattering matrix elements.](image)

**VI.(C): Establishing the existence of universal impedance PDFs for the computer-box enclosure**

Having measured the ensemble of computer-box enclosure scattering matrices $S_{ij}^{\text{cav}}$, and the corresponding radiation-scattering matrix $S_{ij}^{\text{rad}}$ from Sec. IV, we convert these quantities into the corresponding enclosure impedance matrices $Z_{ij}^{\text{cav}}$ and radiation-impedance matrices $Z_{ij}^{\text{rad}}$, respectively using,

$$Z_{ij}^{\text{cav}} = [Z_0]^{1/2} \cdot (1 + S_{ij}^{\text{cav}}) \cdot (1 - S_{ij}^{\text{cav}})^{-1} \cdot [Z_0]^{1/2} \quad \text{and}$$

$$Z_{ij}^{\text{rad}} = [Z_0]^{1/2} \cdot (1 + S_{ij}^{\text{rad}}) \cdot (1 - S_{ij}^{\text{rad}})^{-1} \cdot [Z_0]^{1/2},$$

where the matrix $Z_0$ is a real diagonal matrix whose elements are the characteristic impedances of the transmission lines connected to the driving ports. Here each port has a single operating mode with characteristic impedance of 50 $\Omega$ over the frequency range of the experiment. Each $Z_{ij}^{\text{cav}}$ is then normalized with the corresponding measured $Z_{ij}^{\text{rad}}$ at the same frequency to obtain the normalized impedance matrix $\xi Z$ using Eq.(2).

We consider the experimentally determined normalized impedance $\xi Z$ matrices that lie within a frequency range of 17 to 18 GHz, which is defined as a “data-set”. Each $\xi Z$ matrix in our measured ensemble yields two complex

**VI.(D): Validity of the RCM Induced Voltage Algorithm for the computer-box enclosure**

To test the validity of the RCM induced voltage algorithm for the computer-box enclosure, we first chose an arbitrary frequency range of 4.5 GHz to 5.5 GHz and assume that the losses do not change significantly in this range (i.e. $\alpha$...
is approximately constant). From the $S_{21}^{\text{cav}}(\omega)$ measurements, the typical $Q$ for the computer-box cavity over this frequency range is estimated to be about 45 (i.e., $Q \cong 45$). An estimate of the value of $\alpha = k^2V/(2\pi^2Q)$ using $k = 2\pi f/c$ with $f = 5\text{GHz}$ and $V = 0.38 \times 0.21 \times 0.23 \text{m}^3$ (the physical volume of the computer-box cavity), yields $\alpha \sim 24$. Note that since the computer-box enclosure contains components of different dielectric constants (such as the FR-4 material used to fabricate the motherboard), the electromagnetic-volume of the computer-box enclosure is different from the physical volume. However, since the computer-box enclosure is sufficiently lossy ($\alpha >> 1$), the statistics of the normalized impedance are relatively insensitive to small changes in $\alpha$. This mitigates the effect of errors in the estimate of the enclosure volume.

We then use random matrix Monte Carlo simulations [Appendix A] to generate an ensemble of 100,000 normalized impedance $\tilde{z}$ matrices which correspond to a value of $\alpha = 24$. Combining this ensemble of $\tilde{z}$ matrices with the measured 2x2 radiation impedance matrix $Z_{\text{rad}}^{\text{cav}}$ over the frequency range of 4.5 GHz to 5.5 GHz using Eq.(1), an estimate for the ensemble of the computer-box enclosure impedances in the “Enclosure case” is obtained. In order to determine the nature of the induced voltage PDFs at port-2, two scenarios are simulated by assuming two different functional-forms for the frequency dependence of the radiated power at port-1.

(i) A “Flat-top” (\(\tilde{P}_1(\tilde{f}) = 1\text{Watt}\)) functional-form for the port-1 power, radiated uniformly over the frequency range $f$ from 4.5 GHz to 5.5 GHz (inset of Fig.10(a)).

(ii) A Gaussian-shaped (\(\tilde{P}_1(\tilde{f}) = e^{-(f-\mu)^2/(2\sigma^2)}\text{Watts}\)) functional-form for the port-1 radiated power defined over the frequency range $f$ from 4.5 GHz to 5.5 GHz, which is centered at $\mu = 5\text{GHz}$ and $\sigma = \sqrt{0.025}\text{GHz}$ (inset of Fig.10(b)).

Note: In scenario (i) and (ii), we have assumed that the frequency-dependent port-1 radiated-power is a purely real, scalar quantity. This assumption neglects any phase correlations between the frequency-components of the radiated signal from port-1. We shall also assume that port-2 presents an open-circuit boundary condition ($Z_L = \infty$ in Eq. (4)).

The predicted PDF of the magnitude of the induced voltage at port-2 is shown as the black curve in Fig. 11(a) for the “flat-top” port-1 radiated-power of scenario (i). The black curve in Fig. 11(b) represents the predicted PDF of the magnitude of the induced voltage at port-2 for the Gaussian-shaped port-1 radiated-power of scenario (ii). Note that the induced voltage PDFs in the two scenarios are rather different.

The stars in Fig. 11(a) represent the PDF of the induced voltage at port 2 for the “flat-top” port-1 radiated-power of scenario (i) shown in inset and Eq.(6), where the terms $Z_{11}^{\text{cav}}$ and $Z_{21}^{\text{cav}}$ in Eq. (6) correspond to the experimentally measured Enclosure-Case impedances of the computer-box enclosure. Similarly, the circles in Fig. 11(b) represent the PDF of the induced voltage at port 2 for the Gaussian-shaped port-1 radiated-power of scenario (ii) shown in inset and Eq.(6), where the terms $Z_{11}^{\text{cav}}$ and $Z_{21}^{\text{cav}}$ in Eq. (6) correspond to experimentally measured Enclosure-Case impedances of the computer-box enclosure, and we take $Z_L = \infty$. Good agreement is found between the induced voltage PDFs which were determined numerically (black curves) using only the measured radiation impedance matrix and random matrix Monte Carlo simulations based upon a calculated value of $\alpha$; and those induced voltage PDFs (symbols) which were generated using the experimentally measured enclosure-case impedance matrix ensemble. This confirms the validity of the RCM Voltage Algorithm as an accurate and computationally fast method to predict the statistical nature of induced voltages at a given target-port for a specified excitation at a source-port.

![Fig. 11: (a) Numerically determined PDF of induced voltages at port-2 obtained using the RCM Induced Voltage algorithm for a “flat-top” functional-form of the radiated-power from port-1 (inset) is shown as the black curve. The black stars represent the experimentally derived PDF of induced voltages at port-2 obtained using the elements of the measured enclosure impedance matrix and Eq.(6), for the “flat-top” functional-form of the radiated-power from port-1 (inset). (b) Numerically determined PDF of induced voltages at port-2 obtained using the RCM Induced Voltage algorithm for a Gaussian-shaped frequency-dependence of the radiated-power from port-1 (inset) is shown as the black curve. The black stars represent the experimentally derived PDF of induced voltages at port-2 obtained using the elements of the measured enclosure impedance matrix and Eq.(6), for a Gaussian-shaped frequency-dependence of the radiated-power from port-1 (inset). Good agreement is observed in both cases between the predictions for the induced voltage PDFs obtained from the RCM Induced Voltage algorithm and those generated using the measured enclosure impedance data. In all cases it is assumed that port-2 has open-circuit boundary conditions.](image-url)

**VII. DESIGN GUIDELINES FOR UPSET-RESISTANT GENERIC ENCLOSURE**

Based on the RCM, we can now make suggestions for design of systems that reduce the susceptibility to electromagnetic upset. Some simple design-guidelines for generic complicated enclosures such as computer-boxes, aircraft fuselages, or electronics cabinets, which can be useful in considerations of resilience to upset by an external (or internal) short-wavelength electromagnetic source, are as follows.


a) Increasing the value of the enclosure loss-parameter ($\alpha$): For a three-dimensional enclosure, $\alpha = k^3 V / (2\pi^2 Q)$. Increasing the value of $\alpha$ (e.g., by decreasing the enclosure volume or quality factor $Q$) decreases the fluctuations in the enclosure impedance values [55]. This in turn reduces the probability for large internal field fluctuations, or equivalently large induced voltage fluctuations, on the components housed within the enclosure.

b) Radiation Impedance Engineering:

Perfect coupling implies $Z_{rad}^{cav} = Z_{rad}^{cav}$. Thus creating a large impedance mismatch between the radiation impedance of the port and the characteristic impedance of the transmission lines connected to that port will result in very poor transfer of the incoming electromagnetic energy into the interior of the cavity-enclosure through the port. Apertures, cables, antennas, etc. can be engineered to have a large radiation impedance mismatch at the frequencies of concern. A detailed discussion of how to calculate the port radiation impedance for a large variety of ports is presented in [37].

c) Use of Non-Reciprocal Media:

Though it has not been discussed in this paper, the use of non-reciprocal media such as magnetized ferrites placed within a cavity-enclosure can significantly decrease the amplitude of field intensities. In addition to being inherently lossy (thereby increasing the $\alpha$-value of the cavity-enclosure), non-reciprocal media restrict instances of constructive interference between the rays bouncing within the cavity-enclosure. This in turn reduces the formation of “hot-spots” (regions of high EM field intensities) [56] within the cavity-enclosure.

VIII. ASSUMPTIONS, CAVEATS AND FUTURE WORK

The applicability of RCM is based on certain fundamental assumptions. First, the enclosure has to be substantially large compared to the wavelength of the electromagnetic radiation. This assumption translates into the enclosure supporting many electromagnetic modes (we estimate more than about 50 modes) below the lowest frequency of interest. Second, the enclosure has to display chaotic ray-trajectories. Though this is generally a valid assumption, given the complexity of the internal details in most real-world enclosures such as computer-boxes or aircraft fuselages, we note that some enclosures may show a mixture of chaotic and non-chaotic ray dynamics [25]. Such mixed dynamics are common in complicated enclosures that have several flat metallic surfaces facing each other. Under such conditions, the predictions of RCM may only be partially correct. The presence of flat surfaces within enclosures may also result in “scars” [57, 58, 59], which are modal patterns that exhibit large field intensities near closed ray-trajectories. The presence of scars violates the random plane wave hypothesis within the enclosure, and is currently not treated by the RCM.

The Random Coupling Model uses the radiation impedance matrix to quantify the non-ideal and frequency-dependent coupling between the ports and the enclosure. It has been observed that in the limit where the number of statistically-independent matrices in the ensemble for the enclosure impedance ($Z_{rad}^{cav}$) is infinitely large, the mean value of $Z_{rad}^{cav}$ approaches $Z_{rad}^{rad}$, i.e., $\langle Z_{rad}^{cav} \rangle \approx Z_{rad}^{rad}$ (or equivalently, $\langle S_{rad}^{cav} \rangle \approx S_{rad}^{rad}$) [60, 55, 33]. This observation is of great practical significance in realistic situations where it may not always be possible to accurately determine $Z_{rad}^{rad}$ either numerically or through experimental measurement.

The Random Coupling Model has shown that besides the radiation impedance matrix of the relevant ports, the quantity that determines the shapes and scales of the fluctuations within the enclosure is the loss-parameter $\alpha = k^3 V / (2\pi^2 Q)$. We note that in over-moded enclosures, the $Q$ of the enclosure is frequency-dependent and varies from mode to mode due to the slight differences in the modal patterns near the dissipative structures. However, this variation in $Q$ from mode to mode is small (about 10%) [16] for suitably irregular complicated enclosures. Moreover, most realistic enclosures such as computer boxes, aircraft fuselages or missile casings tend to be of relatively low $Q$ (tens to hundreds). In such cases, the predicted PDFs of the enclosure impedance and scattering quantities are not very sensitive to the loss-parameter $\alpha$. Thus a reasonable estimate of the average $Q$ of the modes around the frequency of interest would suffice in most cases. One possible extension to RCM would be to include the variation of the loss-parameter as a function of frequency. This would be useful when considering complicated enclosures irradiated by wide-band short-wavelength electromagnetic radiation.

Some caveats about the RCM also deserve mention. The RCM is a statistical frequency-domain model and should not be used to predict the outcome of a specific measurement for a specific situation. Also, the presence of parallel reflecting planes within complicated enclosures can lead to mixed-ray dynamics (integrable and non-integrable) and subsequent deviations from RCM predictions. Scars can produce large local enhancements of electromagnetic fields, and their presence and effects fall outside of the Random Coupling Model. One should not use the RCM when the enclosure $Q \sim 1$, or less. In this case there is no reverberation, and the basic assumptions of the model are not satisfied.

Other extensions of the RCM include treatment of transient input waveforms, and treatment of the phases of a broadband signal. The RCM can also be extended to explicitly include short-orbits involving the ports. Short orbits are classical ray trajectories that start from a given port, bounce a small number of times, and then return to the same port, or go on to another port. Effects of these orbits are seen as systematic (non-statistical) variations of the impedance matrix elements as a function of frequency. The RCM has recently been extended to include such orbits [49, 50, 51].
IX. Conclusion

The results discussed in this paper provide experimental evidence in support of the utility of the Random Coupling Model for statistically modeling short-wavelength electromagnetic wave-scattering within three-dimensional complicated enclosures, coupled to multiple ports. The experimental results have shown that the radiation impedance matrix is extremely robust in quantifying the non-ideal port-coupling, even when polarization of the waves and field fluctuations due to the presence of side-walls in the near-field proximity of the driving ports plays a role. We have shown that a minimal amount of information (frequency, volume of the enclosure, typical $Q$ of the enclosure, radiation impedance of the relevant ports, and an estimate of the frequency-dependence of the radiated-power at the source port) is needed to accurately predict the shape and scales of the induced voltages at specific target-ports within large complicated enclosures. Based on the Random Coupling Model, we have also suggested design-guidelines to make a generic three-dimensional complicated enclosure (such as a computer-box or aircraft fuselage) more resistant to upset from an external short-wavelength electromagnetic source.

APPENDIX A

For a wave-chaotic cavity-enclosure filled with reciprocal media and coupled to $N$ single-moded driving ports ($N \geq 1$), the $N \times N$ normalized impedance matrix ($\frac{\xi}{\bar{Z}}$) can be written as,

$$\frac{\xi}{\bar{Z}} = \frac{-j}{\pi} W \frac{1}{\lambda_\infty - j \alpha I} W^T,$$

(A.1)

where the matrix $W$ is a $N \times M$ coupling matrix with each element $W_{nm}$ representing the coupling between the $n^{th}$ driving port ($1 \leq n \leq N$) and the $m^{th}$ eigenmode of the closed cavity ($1 \leq m \leq M$, and $M >> N \geq 1$). Each $W_{nm}$ is assumed to be an independent Gaussian-distributed random number of zero mean and unit variance. The matrix $W^T$ corresponds to the transpose of matrix $W$, and $I$ is a $M \times M$ identity-matrix. The scalar quantity $\alpha$ corresponds to the enclosure loss-parameter and $j = \sqrt{-1}$.

In order to obtain the matrix $\frac{\lambda}{\bar{Z}}$, which is a $M \times M$ diagonal matrix with a set of $M$ $\frac{\lambda}{\bar{Z}}$-values the following procedure is adopted. A real symmetric matrix ($\tilde{H}$) of size $5M \times 5M$ is generated whose elements are independent identically distributed with the on-diagonal elements chosen from a Gaussian distribution of zero-mean and unit-variance, and the off-diagonal elements chosen from a Gaussian distribution of zero-mean and a variance of 0.5. For large $M$ ($M \geq 200$ for the results discussed in this paper), the eigenvalues $\lambda_m$ of $\tilde{H}$ have non-uniform spacing and are distributed as per Wigner’s “Semi-Circle Law” [30] with an average spacing for eigenvalues around the eigenvalue $\lambda$ given by, $\Delta(\lambda) = \pi \sqrt{10M - \lambda^2}$. From this semi-circle distribution, the middle $M$ values are selected and then normalized by multiplying them with $\sqrt{10M / \pi}$, in order to create a sequence of normalized eigenvalues ($\tilde{\lambda}_M$) with an approximately average spacing of unity.

The procedure outlined above results in a single instance of the normalized impedance matrix $\frac{\xi}{\bar{Z}}$. By repeating this procedure about 100,000 times, a sufficiently large ensemble of $\frac{\xi}{\bar{Z}}$ is generated from which its statistical descriptions (PDFs and its moments) are determined.

REFERENCES


