Effects of UHF Stimulus and Negative Feedback on Nonlinear Circuits

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***Abstract*—We investigate the combined effect of rectification and nonlinear dynamics on the behavior of several simple nonlinear circuits. We consider the classic resistor-inductor-diode (*RLD*) circuit driven by a low-frequency (LF) source when an operational amplifier with negative feedback is added to the circuit. Ultra-high-frequency (UHF) signals are applied to the circuit, causing significant changes in the onset of LF period doubling and chaos. Measurements indicate that this effect is associated with a dc voltage induced by rectification of the UHF signal in the circuit. The combination of rectification and nonlinear circuit dynamics produce qualitatively new behavior, which opens up a new channel of radio frequency interference in circuits.**

***Index Terms*—Bifurcations, chaos, diodes, electromagnetic interference (EMI), nonlinear circuits, operational amplifiers, period doubling, radio frequency interference (RFI), rectification.**

I. Introduction

MOST work on electromagnetic interference (EMI) of circuits containing nonlinear elements based on the p-n junction, such as diodes and transistors, has focused on the effect of rectification [1]–[3]. The rectifying nature of the p-n junction causes the envelope of a high-frequency (HF) signal to be stripped off and imposed on the circuit, sometimes with detrimental results. However, nonlinear dynamics, associated for example with the variable capacitance and delayed feedback in the p-n junction, is present in addition to rectification [4]–[10]. Hence, rectification of an envelope signal alone [2] is not the whole story, and nonlinear dynamics of the circuit can introduce important new factors that can enrich the circuit analysis. It remains an open question whether nonlinear dynamics can create qualitatively new circuit behavior in the presence of external stimulus.

The driven resistor-inductor-diode (*RLD*) circuit has been widely investigated because it is the simplest passive nonlinear circuit that displays period doubling and chaos [4]–[10]. Although there is controversy over the precise mechanism of period doubling in this circuit [6], [8]–[12], it has become the standard representation of the nonlinear behavior embodied in p-n semiconductor junctions. However, this simple circuit is not commonly found in isolation in modern circuits. There has not been a systematic investigation of how the nonlinear dynamics of the driven *RLD* circuit is affected when it is embedded in a more complicated circuit topology. In this paper we consider a new and more interesting version of the *RLD* circuit, namely a driven RLD circuit followed by an operational amplifier, the trans-impedance amplifier (TIA). The *RLD*-TIA is a more realistic nonlinear circuit element similar to those found in modern electronics, but it is also simple enough to understand in some detail. We show in this paper that this combination introduces a qualitatively new means of radio frequency interference (RFI) in circuits. We show that high-power ultra HF (UHF) signals enhance the nonlinear and chaotic behavior of the circuit at low frequency (LF), and this comes from a combination of rectification *and* nonlinear circuit dynamics. Hence, the main focus of this work is to investigate two simple nonlinear driven circuits under UHF stimulus and examine the consequences of nonlinear dynamics on circuit behavior.

In Section II, we present the experimental setup, describing the circuits tested, variables measured, components, and parameters used. Section III presents results on circuit phase diagrams to make contact with the classical literature on the driven *RLD* circuit [4]–[9], [12]. Section IV-A discusses the observed chaotic behavior in the circuit under UHF stimulus. We present bifurcation diagrams and give an explanation of the observed effects. In Section IV-B, we investigate, in more detail, the dc voltage generation, comparing the voltages measured from the *RLD* and *RLD*-TIA circuits. In Section IV-C, we present other observed effects from the circuit and discuss the robustness of our results. The conclusion section summarizes the main results.

II. Experiment

There are many ways to investigate chaotic behavior in electronic circuits [14]–[17]. For example, one can examine the state space of variables, such as evolution of the circuit in a two-dimensional space spanned by the electrical current and its time derivative. Also, examination of the time series of voltage and current, bifurcation diagram (constructed using local maxima of a state variable), power spectrum, etc., can describe the dynamics associated with the nonlinear system [13]–[17]. In this paper, we employ bifurcation diagrams constructed from time-series measurements of circuit current and voltage. We also measure the power spectrum of the circuit output voltage to map out the system phase diagram.

When high-power microwave signals are involved, care must be taken during the experimental procedures. The UHF and microwave properties of the circuit are very sensitive to

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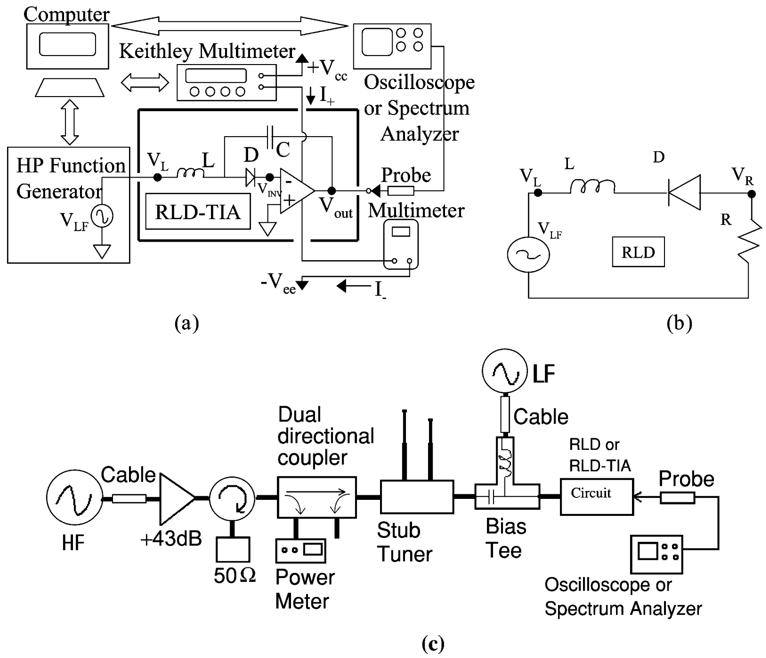

Fig. 1. Experimental setup for HF and LF study.* (a) LF analysis setup for driven *RLD*-TIA. (b) Driven *RLD* circuit schematic. (c) Simultaneous HF and LF stimulus setup used for each circuit. *An HP83620B microwave synthesizer was used as a HF source generator (followed by a + 43-dB LZY-2 minicircuits amplifier), a Weinschel Engineering Model DS-109LL double stub tuner (200–2000 MHz), a Microlab/FXR Model HW-11N Bias Tee (800–2200 MHz), and an HP33120A as the LF voltage source, a Tektronix TDS3052 oscilloscope, a KEITHELEY-196 digital current meter, a FLUKE multimeter, a Tektronix 494P spectrum analyzer, and a PC.

TABLE I
QUANTITIES MEASURED IN EXPERIMENTS

Frequency range	Quantities measured in the RLD circuit	Quantities measured in the RLD-TIA circuit
Low Frequency (1 - 10 MHz)		I_+, I_-, V_{out}, voltage at inverting input of op-amp V_{INV}, DC voltage at inductor V_L. See Figure 1(a).
Low Frequency (1 - 100 MHz)	Voltage drop on resistor V_R, DC voltage at inductor V_L. See Figure 1(b).	
High Frequency (0.75 - 1 GHz)	V_R, V_L, See Figure 1(b),(c).	I_+, I_-, V_{out},V_{INV}, V_L See Figure 1(a),(c).

parasitic reactances. Hence high-impedance and low-capacitance probes have been used to reduce parasitic capacitances from being introduced into the circuit. Since the goal of this work is to investigate simple nonlinear LF driven circuits under UHF stimulus and observe the behavior of period doubling and chaotic onset, a controlled impedance environment was created for the circuit. The circuit was assembled on a 50-Ω characteristic impedance circuit board, and all signals were introduced through 50-Ω transmission lines (see Fig. 1). All driving signals are CW sinusoidal waveforms. Either the *RLD* circuit, [see Fig. 1(b)] or the *RLD*-TIA circuit [see Fig. 1(a)] was the device under test. In the case of the *RLD* circuit, the variable measured was the voltage drop on the resistor, V_R. In the *RLD*-TIA circuit, the op-amp output voltage ($V_{\rm out}$) was the variable measured. We tested circuits with many different parameters (inductors, resistors, and diodes). Tables I and II show those parameters used and variables measured.

A LabView program running in the computer controlled the data acquisition process. In the control program, we select an LF, an initial LF driving voltage, a final driving voltage, and the step voltage to be swept. At each step, a single shot of the oscilloscope screen presenting the voltage output of the circuit was collected and sent to the computer with time series points stored corresponding to approximately 50 cycles of the fundamental LF drive frequency. After the acquisition, we processed the data generating bifurcation diagrams. The bifurcation diagram was obtained from the time series of the variable measured through a program that searches for local maxima (or minima) and plots

TABLE II
PARAMETER VALUES AND COMPONENTS USED IN *RLD* AND *RLD*-TIA CIRCUITS TESTED

Parameters and components	**Values, types for RLD**	**Values, types for RLD-TIA**
R	25 Ω, 100 Ω, 1 kΩ, 10 kΩ, 100 kΩ, 1 MΩ	—
L	150 nH, 390 nH, 10 μH	150 nH, 390 nH, 10 μH
$\mid V_{cc} \mid = \mid V_{ee} \mid$	—	6 V, 8 V, 9 V, 12 V, 15V
Diode	NTE610, 1N5475B	NTE610, 1N5475B, forward and reverse biased
OP-AMP	—	MC1741CU, MC1741SCP1
$C_{feedback}$	—	510 pF

them versus the LF voltage amplitude. Fig. 1(c) shows the experimental setup used to subject the circuits to both LF and HF signals. Table II shows typical parameters used in both circuits. The phase diagrams were obtained by observing the power spectrum of the signal from the circuit and determining either the presence of a subharmonic of the drive frequency signal in the case of period doubling, or by the broadband characteristic of the chaotic spectra in the case of chaos.

To deliver enough HF power to the circuit, we need to impedance match between the source and the circuit. This is accomplished by a stub tuner that is adjusted to maximize the power being delivered to the circuit, and minimize the reflected power. HF and LF signals are applied to the circuit through a bias tee to prevent the HF component from being delivered to the LF source generator and *vice-versa*. The HFs applied vary from 750 MHz to 1 GHz and power from 0 dBm to + 43 dBm. LF signals vary from 1 to 100 MHz and power from − 10 dBm to + 33 dBm.

III. PHASE DIAGRAM OF RLD AND RLD-TIA CIRCUITS

It is helpful to first establish the regions of parameter space where period doubling and chaos occur. In the circuits of interest to us here, the important parameters (besides the circuit element values) are the driving frequency and amplitude [12]. The phase diagrams presented in Fig. 2 indicate the presence of period doubling or chaos in the LF drive amplitude versus LF drive frequency plane. The "U" shape of this diagram indicates that either period doubling or chaos are more easily created at frequencies near the low-amplitude resonant frequency of the circuit, as has been established for a wide variety of *RLD* circuits [10], [12]. The existence of period doubling and chaos depends mainly on the nonlinear capacitance and the relative size of the drive period and the reverse recovery time of the diode [10]. The low-amplitude resonant frequency (f_0) is estimated as $1/2\pi\sqrt{\mathrm{LC}_0}$, where C_0 is the transition capacitance of the diode at zero voltage bias [4], [5], [12]. We observe that for the case of the *RLD* circuit ($L = 390$ nH, $R = 25\Omega$, NTE610 Diode with measured $C_0 = 16$ pF, yielding an estimated $f_0 \approx 64$ MHz) only period doubling is observed, and not chaos, for the frequency range 63–83 MHz in Fig. 1(b); see Fig. 2(a). We also applied high-power UHF up to + 41 dBm to this *RLD* circuit, but there was no change in the onset of period doubling for Fig. 2(a).

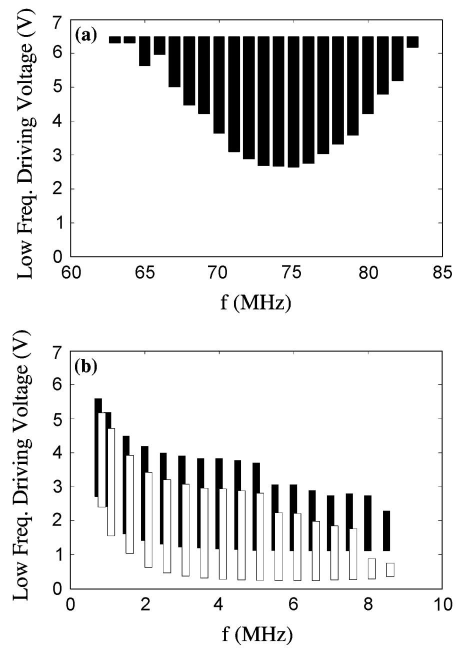

Fig. 2. Period-doubling phase diagrams. (a) Driven *RLD* circuit, where only period doubling is observed (maximum voltage applied was 6.5 V to prevent damage to the circuit components). (b) *RLD*-TIA where both chaos and period doubling are observed (maximum voltage applied was 10 V). Black bars were obtained with no HF signal applied. White bars were obtained with a HF signal applied ($f = 767$ MHz, power = + 33 dBm). The bars represent parameter values where either period doubling or chaos was observed. Note that the white and black bars perfectly overlap in (a), meaning that the UHF stimulus has no effect on the LF period doubling in the *RLD* circuit.

If we add the TIA [Fig. 1(a)] with (capacitive) negative feedback ($C_{\mathrm{feedback}} = 510$ pF, MC1741CU op-amp) to the *RLD* circuit, we see that the nonlinear behavior shifts to a lower frequency range, 1–6.5 MHz [see Fig. 2(b)—black bars]. This is expected because the feedback capacitor acts as if it is in parallel with the varactor diode, giving $C_{\mathrm{total}} \approx C_{\mathrm{feedback}} + C_0$, yielding an estimated $f_0 \approx 11$ MHz. As with the *RLD* circuit, we expect to find period doubling in the vicinity of this new resonant frequency [10], [12]. In this case, not only period doubling was seen but also chaos. The appearance of chaos with the addition of the TIA is most likely associated with the movement of the circuit resonant frequency from $f_0 > 1/\tau_{RR}$ (*RLD*) to $f_0 < 1/\tau_{RR}$ (*RLD*-TIA), where $\tau_{RR} \approx 45$ ns is the reverse recovery time of the NTE610 diode [10]. From Fig. 2, we see that the addition of the TIA feedback to the *RLD* circuit in this particular case effectively enhances the parameter range over which period doubling is observed at LF, although this is by no means a general result. Prior work has shown that chaos in the driven *RLD* circuit is a nonlinear function of many parameters, including signal amplitude, duty cycle, dc offset, frequency, etc., [10], [12]. Embedding the *RLD* circuit in a more complicated circuit topology can be expected to produce many complications, one of which is discussed in detail below.

IV. UHF Stimulus

A. Enhancement of Chaotic Behavior

It was seen by Oksasoglu and Vavriv, and Shygimaga *et al.* that a nonlinear circuit driven by two properly chosen signals may present chaotic instabilities at much lower voltage levels than with one signal alone [18], [19]. They showed that chaotic behavior can occur when the "HF" signal is near the resonant frequency of the circuit and the "LF" signal is on the order of the 3-dB bandwidth. Our investigation considers a LF mode of the order of the resonant frequency of the circuit, while the HF mode is in the UHF range, about 10^3 times higher. We are motivated to use this combination by our interest in the combined effects of rectification and nonlinear dynamics, and by the possible generalization of Oksasoglu and Vavriv and Shygimaga *et al.*'s results [18], [19].

The general result of adding UHF stimulus to the circuit is shown in Fig. 2(b) (white bars). The UHF causes the *RLD*-TIA system to show period doubling and chaos at a significantly lower frequency driving voltage over the entire frequency range. This unusual effect has not been documented before, to our knowledge. To investigate further, bifurcation diagrams of the driven *RLD*-TIA circuit are shown in Fig. 3(a)–(d) as a function of UHF stimulus power. As the incident UHF power is increased, the nonlinear response at LF is enhanced, causing the onset of period doubling and chaos to begin at the lower frequency driving voltages. Note that each subsequent bifurcation diagram reproduces all the features of the others at higher LF driving amplitude, to good approximation. This is remarkable because, naively, we would expect a shift of the bifurcation diagrams by an amount equal to the rectified dc voltage. Instead, the effect of the UHF is to "uncover" more and more of the underlying bifurcation diagram. It seems that the UHF "tickles" the system out of period 1 behavior into more complicated behavior.

Tanaka *et al.* observed that the driven *RLD* circuit presents sheet structures in a global bifurcation diagram built using a three-dimensional state space [20]. The first two state variables are those shown in the phase diagram of Fig. 2, and the third variable used there is S/T, where S represents the time the trajectory in the state space spends in the diffusion capacitance region of the diode (C_d), while T stands for the time it spends in the transition capacitance (C_t) region [20] (see Fig. 4). Our results resemble theirs, where the third dimension in our case can be explored through the HF power. Hence the "uncovering" feature observed in Fig. 3(a)–(d) may be a consequence of jumping over the bifurcation sheet structures as we change the UHF power.

In order to understand the UHF power dependence of the *RLD*-TIA circuit, we turned off the UHF signal and added a dc offset voltage in series with the LF source generator (before the bias tee). The resulting bifurcation diagrams are presented in Fig. 3(e)–(h) for several values of applied dc offset voltage. If we compare those results to Fig. 3(a)–(d), we see nearly identical behavior in the bifurcation diagrams, suggesting that the UHF signal is equivalent to a dc offset voltage applied to the diode and op-amp. This can be understood as follows. As the forward biased dc offset voltage across the diode is increased we are getting closer to the very nonlinear region of the model diode curve $C(V_{\mathrm{LF}})$ (see Fig. 4) [21]. Hence, only relatively small amplitudes of LF drive voltage are required to reach that region. We tested this hypothesis by applying a negative bias to the diode and we observed the opposite behavior, i.e., a "re-covering" of the bifurcation diagram forcing us to apply a larger LF driving voltage to see period doubling and chaos in the circuit.

We repeated this experiment for the *RLD* circuit using parameters and conditions where LF chaotic behavior is present (e.g., $L = 10\ \mu$H, $R = 25\ \Omega$, and NTE610 diode, $f_0 \approx 12.6$ MHz, close to that of the *RLD*-TIA circuit), and we did not observe significant changes in the onset of chaos due to the applied UHF.

B. Measurement of Rectified dc Offset Voltage

In the Section IV-B, we observed that the nonlinear properties of the *RLD*-TIA circuit can be modified if we apply a tone with a frequency much greater than the natural resonant frequency of the circuit. Specifically, the UHF component seems to be equivalent to a dc source voltage when the operational amplifier adds a nonlinear feedback in the *RLD* circuit. Therefore, we did an experiment to measure the dc voltage level obtained as a result of the UHF stimulus. Fig. 5 shows the dc voltage offset measured at the inductor [V_L in Fig. 1(a) and (b)] using the setup of Fig. 1(c) where the circuit inside the box was either the *RLD* or the *RLD*-TIA. The LF source generator was replaced by a digital multimeter to measure the dc voltage level through the bias tee. We observe that the voltage levels developed in the *RLD*-TIA are much higher than in the case of the *RLD* circuit for the same HF power (even using other values of resistance, see Table II). These results, together with the observations from the previous section, suggest that the negative feedback and the UHF stimulus cause the circuit to develop a dc voltage level at the inductor, [1] thus bringing the circuit to a region where the diode capacitance is more nonlinear (see Fig. 4), resulting in the onset of chaotic behavior at lower values of the LF drive voltage.

C. Other Observations

In addition to the observations made above, unusual behavior at period doubling and chaotic transitions of the *RLD*-TIA circuit were found. We observed discontinuities in the dc supply

[1] Observe that the signs of the dc offset voltages in Fig. 3 and curve (D) of Fig. 5 are opposite, but reflect the fact that the dc current traversing the forward biased diode in both cases is positive. Note that for the curve (D) of Fig. 5, we replaced the LF + dc source generator with a digital multimeter that measures the voltage drop at the bias tee capacitor. Thus, when we apply the HF signal, the diode acts like a battery and the positive current charges this capacitor with a negative dc voltage when measured from the bias tee inductor. To obtain an equivalent current through the diode in the absence of HF, one must supply a positive voltage through the bias tee.

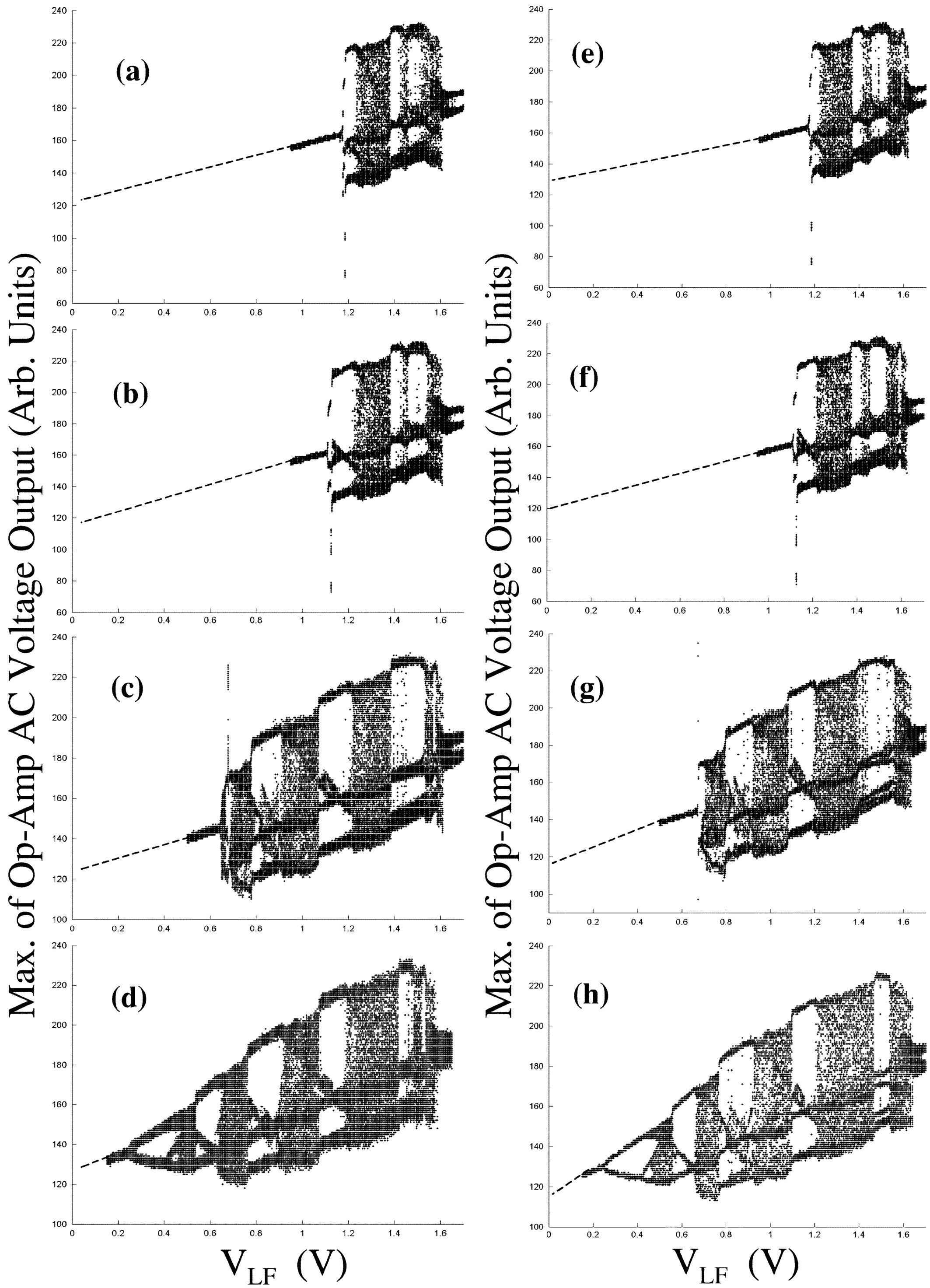

Fig. 3. LF bifurcation diagrams for RF stimulus (left column) and dc voltage offset (right column) of the *RLD*-TIA circuit. All graphs show the relative maximum of the op-amp output voltage (using 50 cycles of the driven voltage) plotted versus the amplitude of the LF driving voltage. (a) No HF power. (b) $P_{\rm HF} = +20$ dBm. (c) $P_{\rm HF} = +30$ dBm. (d) $P_{\rm HF} = +40$ dBm RF stimulus. (e) DC offset $= 0$. (f) DC offset $= +40$ mV. (g) DC offset $= +300$ mV. (h) DC offset $= +540$ mV. (Parameters: $L = 150$ nH, NTE610 Diode forward biased, RF $= 800$ MHz, LF $= 5.5$ MHz, op-amp MC1741CU). Dashed lines indicate where period-1 behavior is observed.

current to the op-amp at each period doubling transition and chaotic transition. Van Buskirk and Jeffries observed that the bifurcation diagram of the average current in the driven RLD circuit also presents discontinuities at transitions between periodic and chaotic behavior [7]. However, the calculated dissipated power associated with the supply current discontinuities is far too small to create irreversible changes in the op-amp by thermal means. In no case have we ever observed failure or other irreversible changes of the op-amp under any experimental circumstances considered thus far.

PSPICE simulations were done for the *RLD* circuit. We have qualitative agreement, for example, with Fig. 2(a), although

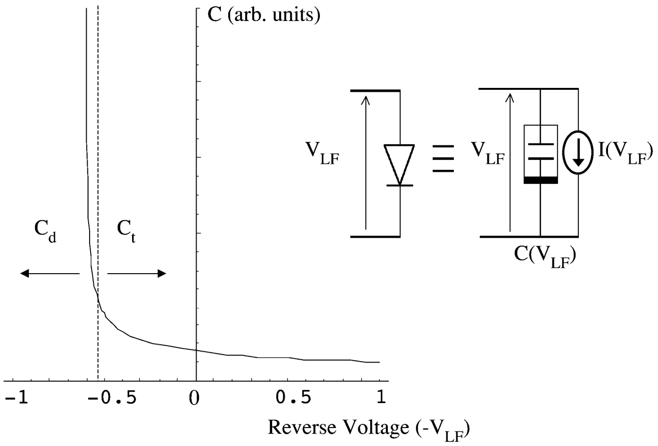

Fig. 4. General schematic of the diode circuit model. The crossover between transition capacitance C_t and diffusion capacitance C_d occurs when the charge on the "capacitor plate" goes through zero [5], [20]. The forward dc offset voltage brings the circuit operation to a region close to where $C(V_{\mathrm{LF}})$ is very nonlinear. $I(V_{\mathrm{LF}})$ is the nonlinear diode current-voltage relationship [4], [5].

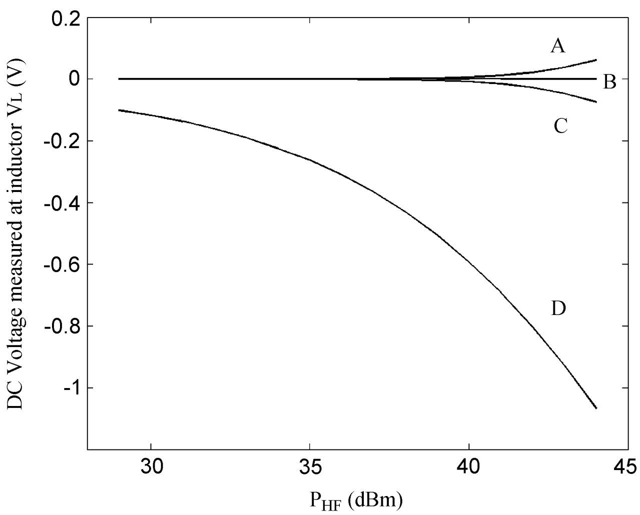

Fig. 5. DC offset voltage measured at the inductor versus UHF stimulus power: $L = 390$ nH, NTE610 Diode, HF$= 800$ MHz. *RLD* circuit: $R = 25\Omega$. *RLD*-TIA circuit: op-amp MC1741CU. (A) Diode reverse biased — *RLD*. (B) Diode reverse biased — *RLD*-TIA. (C) Diode forward biased — *RLD*. (D) Diode forward biased — *RLD*-TIA.

the simulation shows period doubling for frequencies below 60 MHz and above 85 MHz. Nevertheless the "U" shape is clearly observed, and the voltage scales are similar. More extensive modeling of the driven *RLD* circuit is presented in [10]. We did not observe period doubling or chaos for the *RLD*-TIA circuit using the 741 op-amp model furnished with SPICE. A more sophisticated transistor-level model may be required to reproduce the observed behavior.

Also, the positive dc level branch measured for a reverse-biased diode in the RLD circuit (curve A in Fig. 5) is not observed for the reverse-biased diode with the operational amplifier present (*RLD*-TIA circuit) (curve B in Fig. 5). Hence diode bias symmetry is not preserved when an operational amplifier with negative feedback is present, and this explains why we need to have the diode forward biased in the circuit [see Fig. 1(a)] to observe nonlinear behavior.

Although most of the results presented here were obtained using the NTE610 diode and 150- and 320-nH inductors in the *RLD*-TIA circuit, we also observed similar results for the varactor diode 1N5475B, suggesting that the enhancement in the nonlinear behavior is a general property. For example, the results in Fig. 3 were reproduced using $L = 10\ \mu$H and $L = 320$ nH with the NTE610 diode. The results of Figs. 3 and 5 were also obtained with an 1N5475B diode and $L = 150$ nH using LF $= 1.9$ MHz and HF $= 767$ MHz, and also when the op-amp was changed to an MC1741SCP1. Also, variations in V_{cc} and V_{ee} applied to the op-amps do not change our results. The results in Fig. 5 for the *RLD* case are almost the same when *R* is changed from 25 Ω to 1 kΩ to 20 kΩ, or to 1 MΩ. Finally, a more detailed investigation of the origins of chaos in the driven *RLD* circuit is presented in [10].

V. Conclusion

We have examined the canonical *RLD* circuit modeling the nonlinear dynamics of a p-n junction, along with a TIA, and found a striking change in the onset of period doubling under UHF stimulus. Phase and bifurcation diagrams were presented showing that UHF power enhances the nonlinear and chaotic behavior at lower LF voltages for the *RLD*-TIA circuit. This is associated with a dc voltage induced by rectification in the circuit, which was verified by direct measurement in both the *RLD* and *RLD*-TIA circuits. The combination of rectification and nonlinear characteristics can thus enhance the range of chaotic behavior of the circuit. Rectification is only part of the effect of a UHF stimulus signal on a nonlinear circuit. The nonlinear dynamics of the circuit can greatly enhance the disruptive effect of the UHF stimulus. This suggests that qualitatively new behavior may be found when p-n junctions are embedded in modern electronics.

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References

[1] R. E. Richardson, V. G. Puglielli, and R. A. Amadori, "Microwave interference effect in bipolar transistors," *IEEE Trans. Electromagn. Compat.*, vol. EMC-17, pp. 216–219, Nov. 1975.
[2] C. E. Larson and J. M. Roe, "A modified Ebers-moll transistor model for RF-interference analysis," *IEEE Trans. Electromagn. Compat.*, vol. EMC-21, pp. 283–290, Nov. 1979.
[3] R. E. Richardson, "Modeling of low-level rectification RFI in bipolar circuitry," *IEEE Trans. Electromagn. Compat.*, vol. EMC-21, pp. 307–311, Nov. 1979.
[4] P. S. Linsay, "Period doubling and chaotic behavior in a driven harmonic oscillator," *Phys. Rev. Lett.*, vol. 47, no. 19, pp. 1349–1352, 1981.
[5] D. S. Brorson, D. Dewey, and P. S. Linsay, "Self-replicating attractor of a driven semiconductor oscillator," *Phys. Rev. A*, vol. 28, pp. 1201–1203, 1983.
[6] T. Matsumoto, L. O. Chua, and S. Tanaka, "Simplest chaotic nonautonomous circuit," *Phys. Rev. A*, vol. 30, pp. 1155–1157, 1984.
[7] R. Van Buskirk and C. Jeffries, "Observation of chaotic dynamics of coupled nonlinear oscillators," *Phys. Rev. A*, vol. 31, pp. 3332–3357, 1985.
[8] Z. Su, R. W. Rollins, and E. R. Hunt, "Simulation and characterization of strange attractors in driven diode resonator systems," *Phys. Rev. A*, vol. 40, pp. 2698–2705, 1989.

[9] D. M. Vavriv and A. Oksasoglu, “Stability of varactor circuits,” *Electron. Lett.*, vol. 30, pp. 462–463, 1994.
[10] R. M. de Moraes and S. M. Anlage, “Unified model and reverse recovery nonlinearities of the driven diode resonator,” *Phys. Rev. E*, vol. 68, pp. 026 201-1–026 201-9.
[11] C.-M. Kim, C.-H. Cho, C.-S. Lee, J.-H. Yim, J. Kim, and Y. Kim, “Period doubling and bifurcation in an electronic circuit with a fast-recovery diode and square-wave input,” *Phys. Rev. A*, vol. 38, pp. 1645–1648, 1988.
[12] T. L. Carroll and L. M. Pecora, “Parameter ranges for onset of period doubling in the diode resonator,” *Phys. Rev. E*, vol. 66, pp. 046 219-1–046 219-8, 2002.
[13] E. Ott, *Chaos in Dynamical Systems*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 2002.
[14] *Special Issue on Chaos in Nonlinear Electronics Circuits—A, IEEE Trans. Circuits Syst. I*, Oct. 1993, vol. 40, pp. 637–788.
[15] *Special Issue on Chaos in Nonlinear Electronics Circuits—B, IEEE Trans. Circuits Syst. I*, Nov. 1993, vol. 40, pp. 789–884.
[16] *Special Issue on Chaos in Nonlinear Electronics Circuits—C, IEEE Trans. Circuits Syst. II*, Oct. 1993, vol. 40, pp. 593–704.
[17] T. Carroll and L. M. Pecora, *Nonlinear Dynamics in Circuits*. Singapore: World Scientific, 1995.
[18] A. Oksasoglu and D. M. Vavriv, “Interaction of low- and high-frequency oscillations in a nonlinear *RLC* circuit,” *IEEE Trans. Circuits Syst. I*, vol. 41, pp. 669–672, Oct. 1994.
[19] D. V. Shygimaga, D. M. Vavriv, and V. V. Vinogradov, “Chaos due to the interaction of high- and low-frequency modes,” *IEEE Trans. Circuits Syst. I*, vol. 45, pp. 1255–1260, Dec. 1998.
[20] S. Tanaka, S. Higuchi, and T. Matsumoto, “Sheet structure in global bifurcations of a driven R-L-diode circuit,” *Phys. Rev. E*, vol. 54, pp. 6014–6028, 1996.
[21] A. S. Maksimov and N. A. Maksimov, “Dynamics of a nonlinear oscillatory circuit with a p-n junction at various bias voltages and under the action of an external signal,” *Sov. Phys. Tech. Phys.*, vol. 34, pp. 939–940, 1989.

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