Fundamentals of Microwave Superconductivity

Short Course Tutorial

Superconductors and Cryogenics in Microwave Subsystems

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Objective

To give a basic introduction to superconductivity, superconducting electrodynamics, and microwave measurements as background for the Short Course Tutorial "Superconductors and Cryogenics in Microwave Subsystems"

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Outline

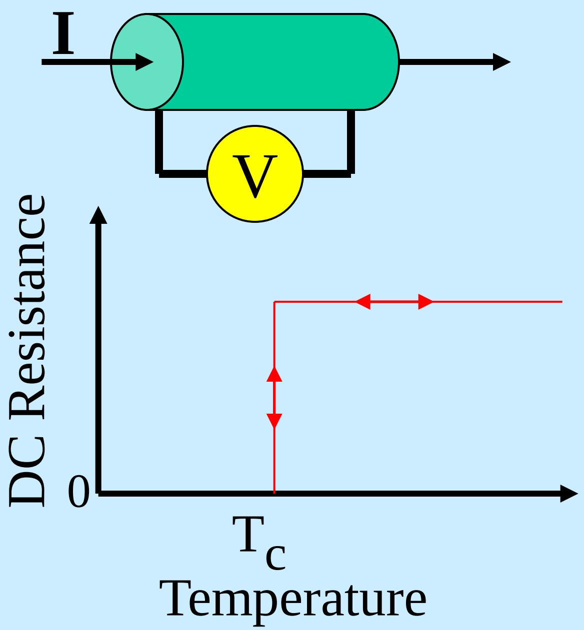
- Superconductivity
- Microwave Electrodynamics of Superconductors
- Experimental High Frequency Superconductivity
- Current Research Topics
- Further Reading

Superconductivity

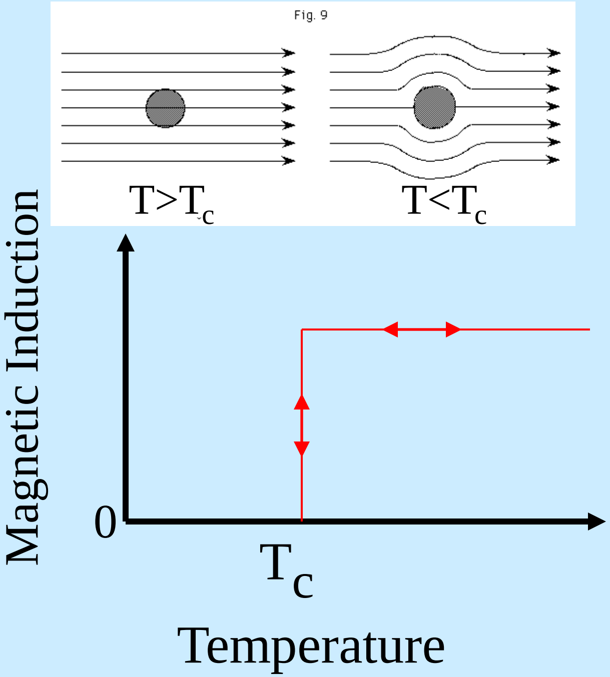
- **The Three Hallmarks of Superconductivity**
- **Superconductors in a Magnetic Field**
- **Where is Superconductivity Found?**
- **BCS Theory**
- **High-T_c Superconductors**
- **Materials Issues for Microwave Applications**

The Three Hallmarks of Superconductivity

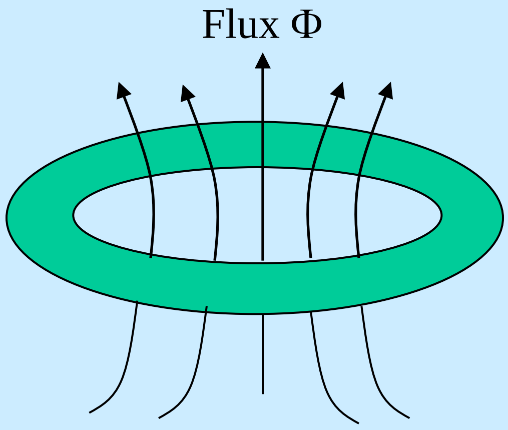
Zero Resistance



Complete Diamagnetism



Macroscopic Quantum Effects



Flux quantization $\Phi = n\Phi_0$
Josephson Effects

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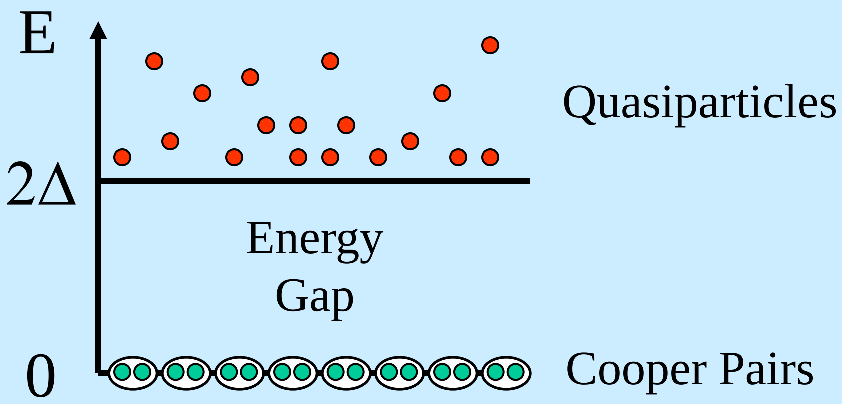
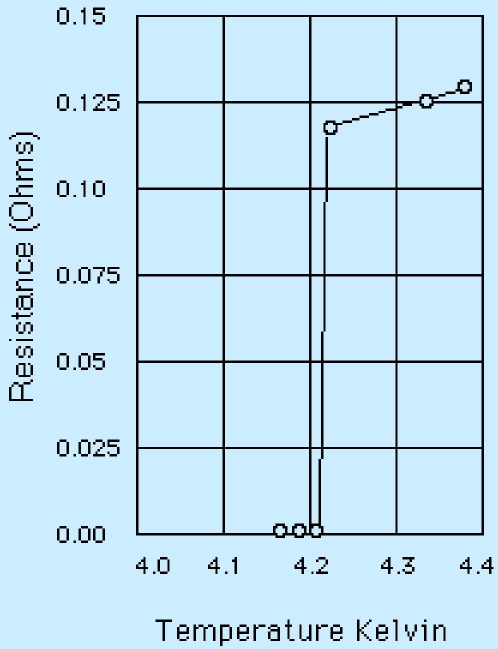
Zero Resistance

R = 0 only at ω = 0 (DC)

R > 0 for ω > 0





The Kamerlingh Onnes resistance measurement of mercury. At 4.15K the resistance suddenly dropped to zero

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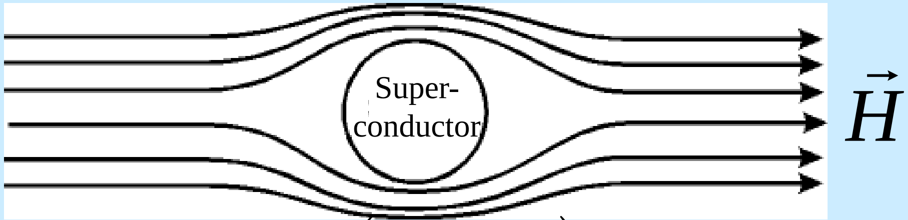
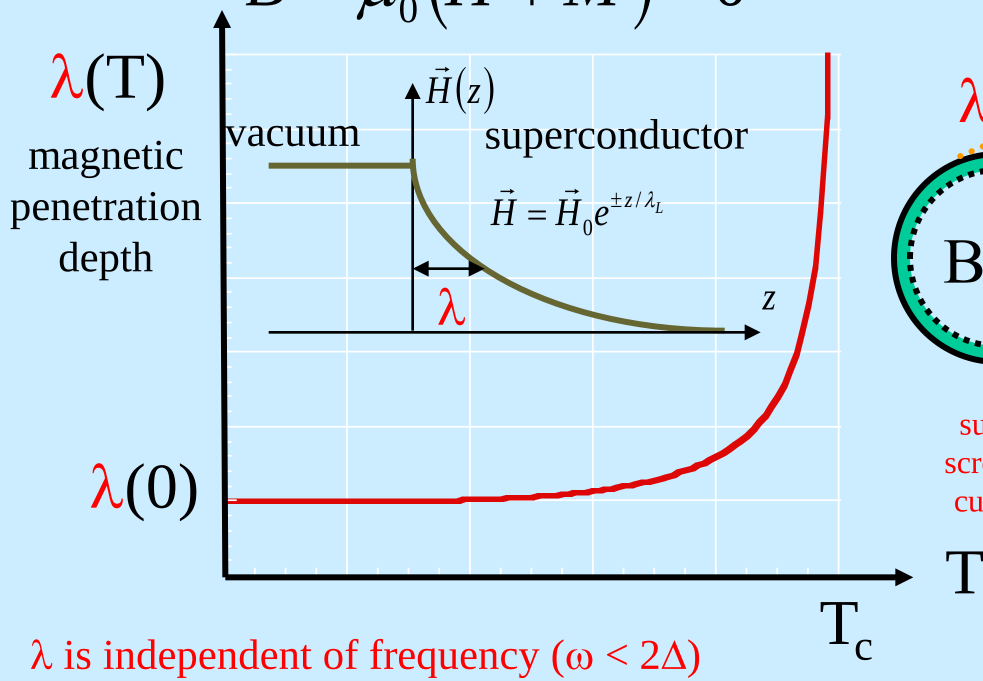
Perfect Diamagnetism

Magnetic Fields and Superconductors are not generally compatible

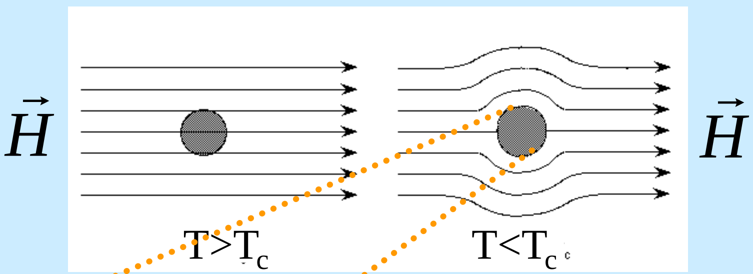


$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = 0$$

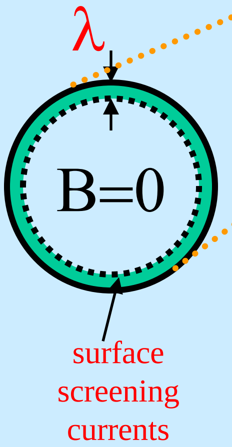


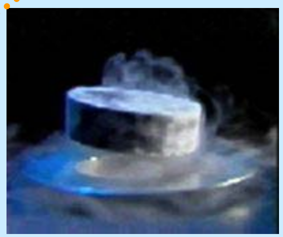
λ is independent of frequency ($\omega < 2\Delta$)

The Meissner Effect



Spontaneous exclusion of magnetic flux





The Yamanashi MLX01 MagLev test vehicle achieved a speed of 343 mph (552 kph) on April 14, 1999

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Macroscopic Quantum Effects

Superconductor is described by a single Macroscopic Quantum Wavefunction

$$\Psi = |\Psi| e^{i\phi}$$

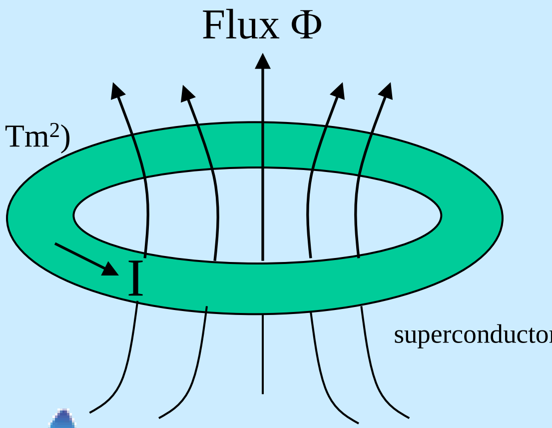
Consequences:

Magnetic flux is quantized in units of $\Phi_0 = h/2e$ (= 2.07 x 10^{-15} Tm^2)

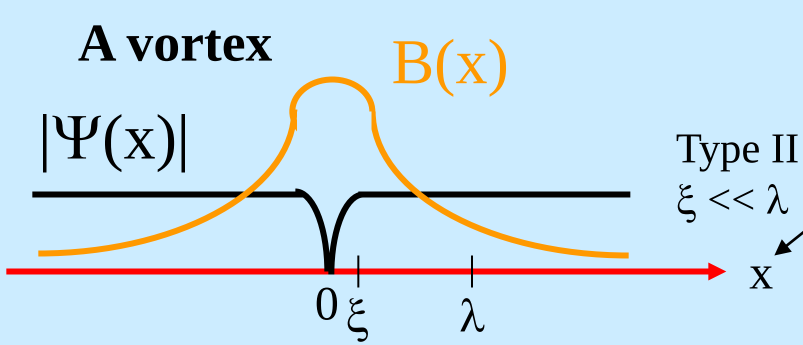
R = 0 allows persistent currents

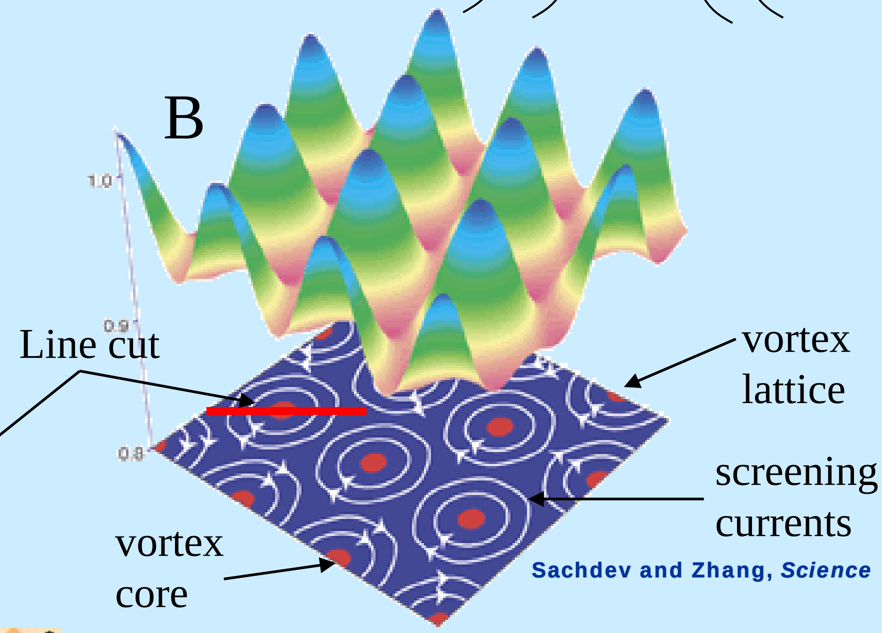
Current I flows to maintain $\Phi = n\,\Phi_0$ in loop

n = integer, h = Planck's const., 2e = Cooper pair charge



Magnetic vortices have quantized flux





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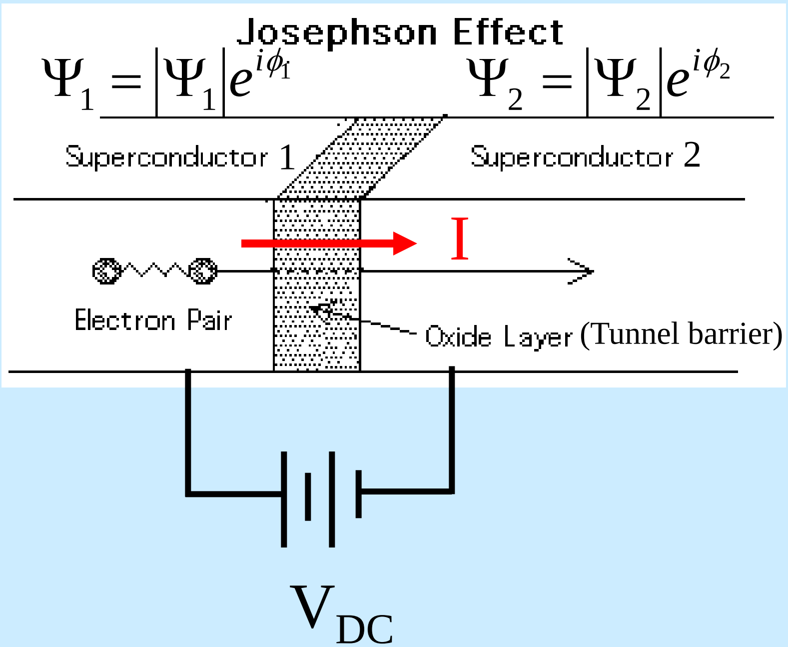
Macroscopic Quantum Effects Continued

Josephson Effects (Tunneling of Cooper Pairs)



$$I = I_c \sin(\phi_1 - \phi_2)$$

DC Josephson Effect

$$(\dot{\phi}_1 - \dot{\phi}_2) = \frac{e^* V_{DC}}{\hbar}$$

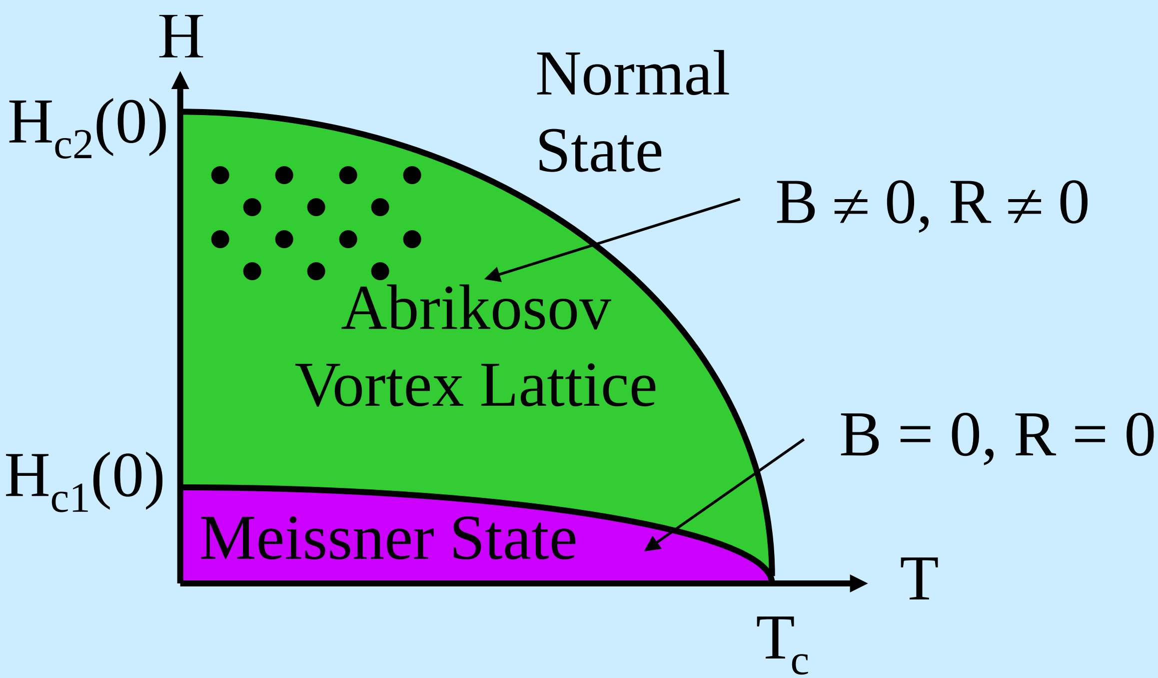
$$I = I_c \sin\left(\frac{e^* V_{DC}}{\hbar} t + \phi_0\right)$$

AC Josephson Effect

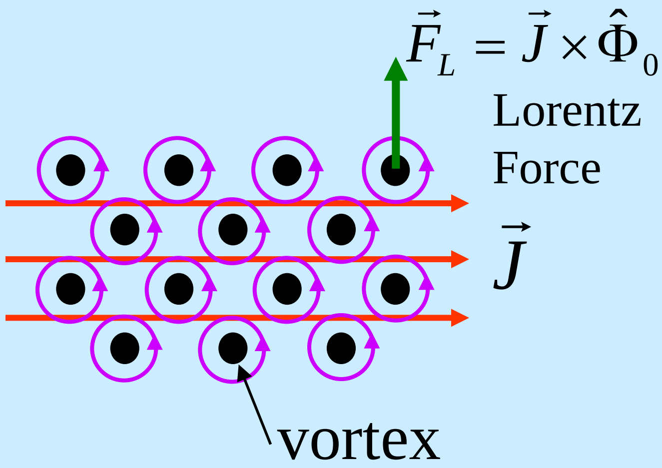
Quantum VCO: $\dfrac{e^*}{h} = \dfrac{1}{\Phi_0} = 483.593420 \dfrac{\text{MHz}}{\mu\text{V}}$

Superconductors in a Magnetic Field

The Vortex State



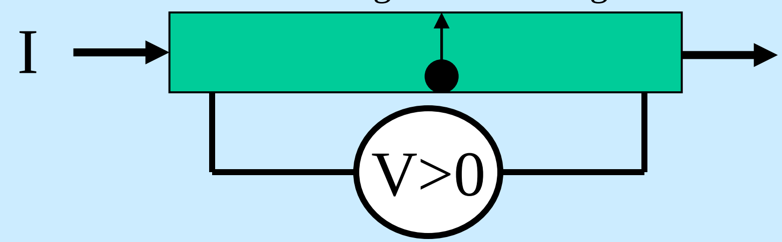
Type II SC



Vortices also experience a viscous drag force:

$$\vec{F}_{Drag} = -\eta \vec{v}_{vortex}$$

Moving vortices create a longitudinal voltage



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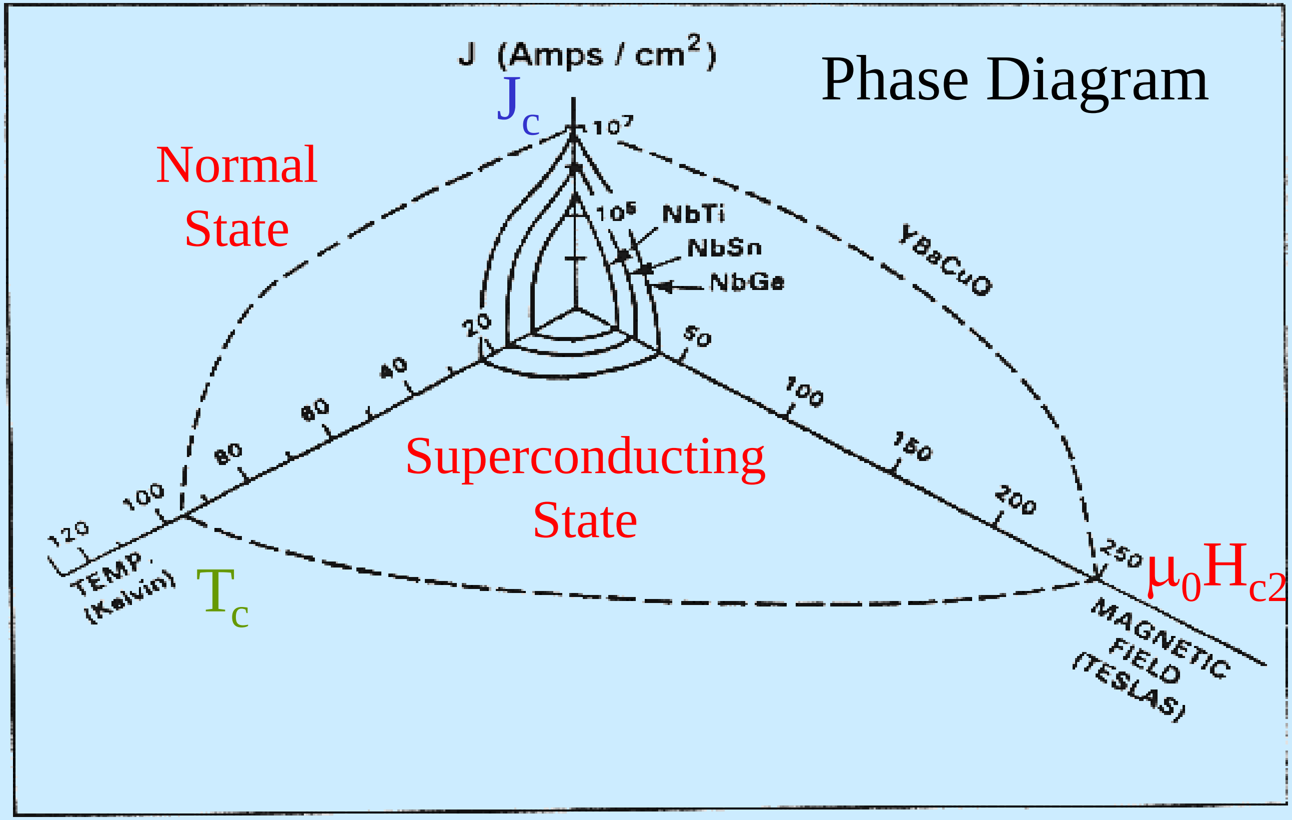
What are the Limits of Superconductivity?



$$f_{super} = f_{normal} + \alpha(T)|\psi|^2 + \frac{\beta(T)}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)\psi\right|^2 + \frac{\mu_0 h^2}{2}$$

Ginzburg-Landau free energy density

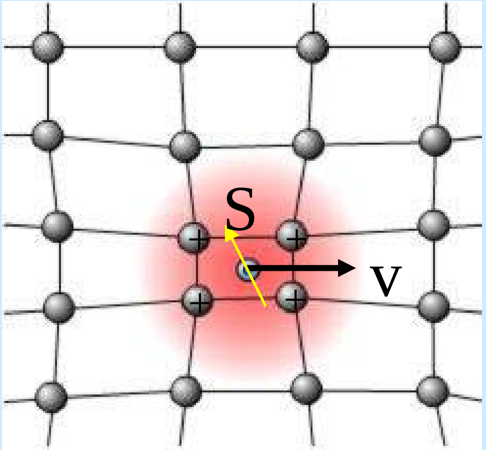
Temperature dependence

Currents

Applied magnetic field

BCS Theory of Superconductivity

Bardeen-Cooper-Schrieffer (BCS)

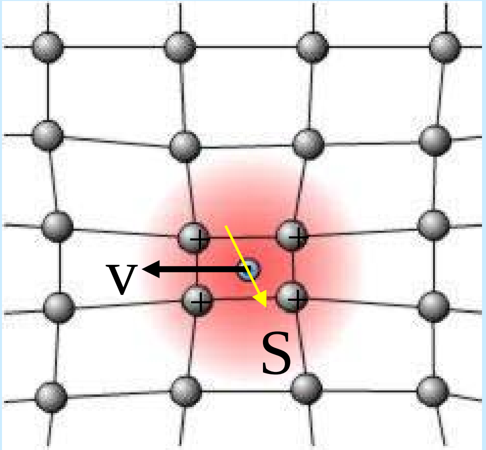


First electron polarizes the lattice

Cooper Pair

s-wave ($\ell = 0$) pairing

Spin singlet pair



Second electron is attracted to the concentration of positive charges left behind by the first electron

$$T_c \cong \Omega_{Debye}\, e^{-1/NV}$$

Ω_{Debye} is the characteristic phonon (lattice vibration) frequency
N is the electronic density of states at the Fermi Energy
V is the attractive electron-electron interaction

A many-electron quantum wavefunction Ψ made up of Cooper pairs is constructed with these properties:

An energy $2\Delta(T)$ is required to break a Cooper pair into two quasiparticles (roughly speaking)

Cooper pair size: $\xi = v_F \cdot \frac{\hbar}{\Delta}$

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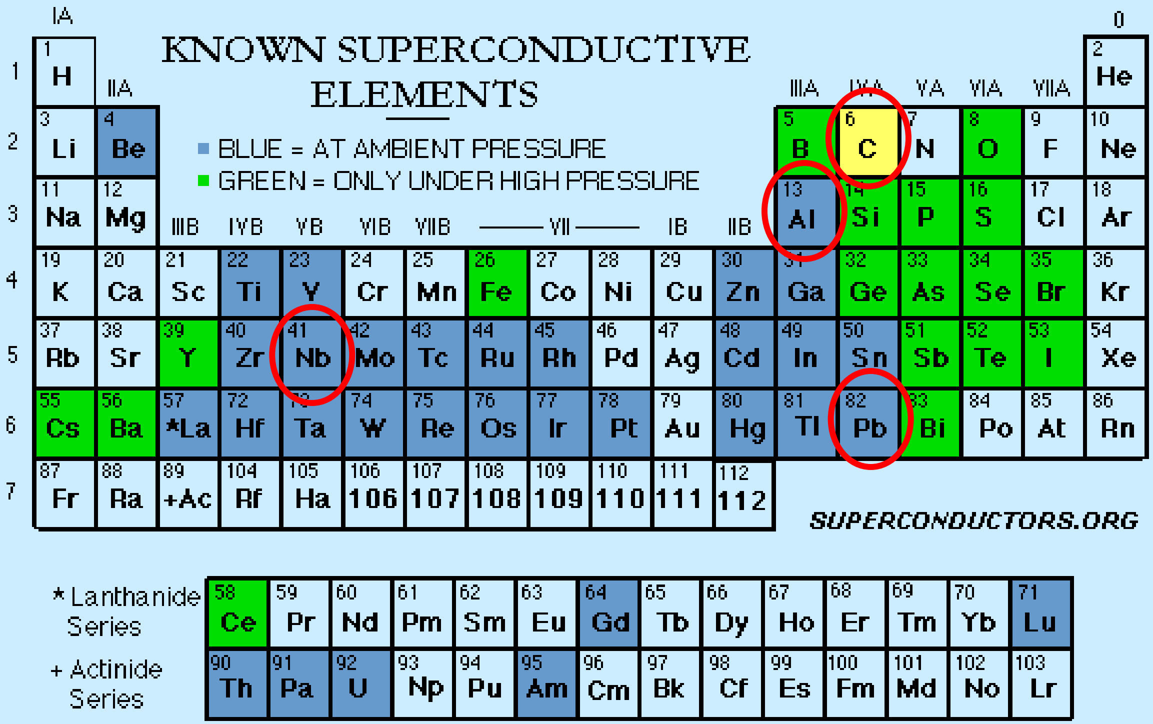
http://www.chemsoc.org/exemplarchem/entries/igrant/hightctheory_noflash.html

Where do we find Superconductors?



Also:

Nb-Ti, Nb_3Sn, A_3C_{60}, NbN, MgB_2, Organic Salts ($(TMTSF)_2X$, $(BEDT\text{-}TTF)_2X$), Oxides (Cu-O, Bi-O, Ru-O,…), Heavy Fermion (UPt_3, $CeCu_2Si_2$,…), Electric Field-Effect Superconductivity (C_{60}, $[CaCu_2O_3]_4$, plastic), …

Most of these materials, and their compounds, display spin-singlet pairing

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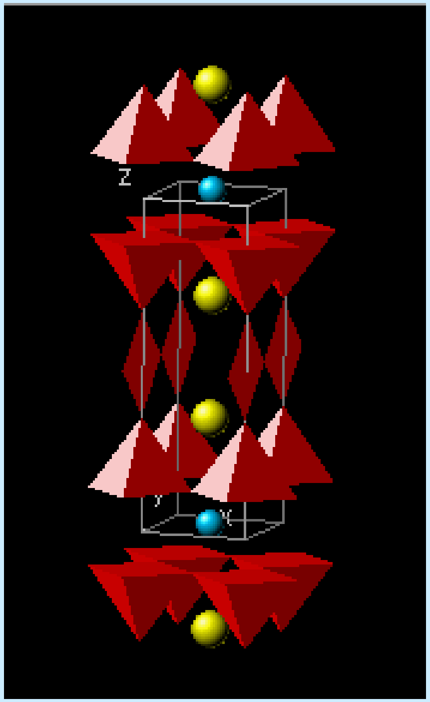
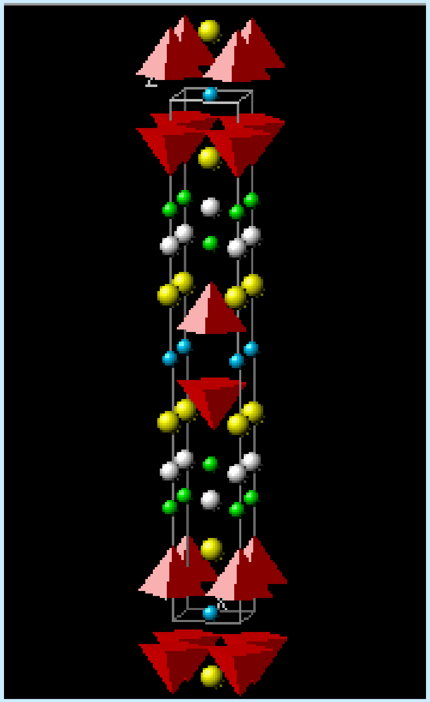
The High-T_c Cuprate Superconductors

Layered structure – quasi-two-dimensional
Anisotropic physical properties
Ceramic materials (brittle, poor ductility, etc.)
Oxygen content is critical for superconductivity

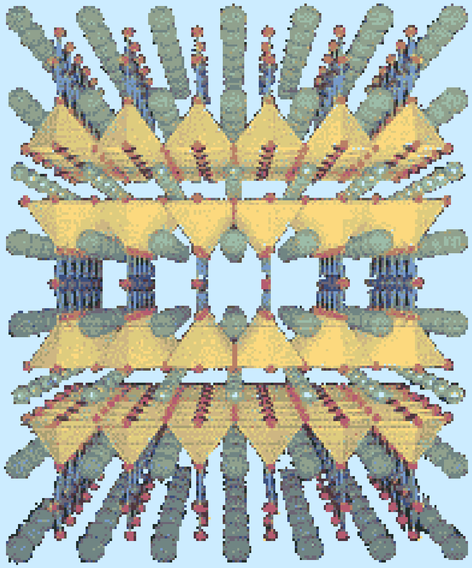
Spin singlet pairing
d-wave ($\ell = 2$) pairing



$YBa_2Cu_3O_{7-\delta}$



$Tl_2Ba_2CaCu_2O_8$



Two of the most widely-used HTS materials in microwave applications

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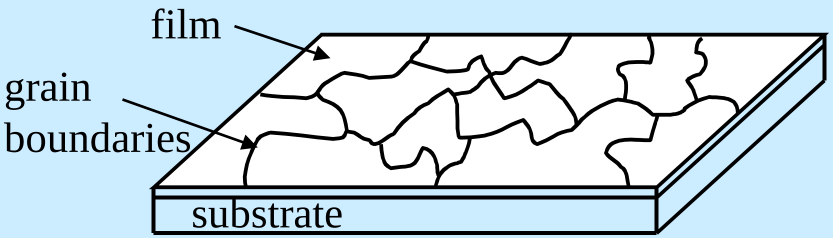
HTS Materials Issues Affecting Microwave Applications



Most HTS materials made as epitaxial thin films for use in planar microwave devices

High-T_c ➔ small Cooper Pair size (ξ – correlation length)

$$\xi = v_F \cdot \frac{\square}{\Delta} \propto v_F \cdot \frac{1}{T_c}$$

$\xi \sim 1 – 2$ nm for HTS materials used in microwave applications

Superconducting pairing is easily disrupted by defects:

- grain boundaries
- cracks

Josephson weak links are created, leading to:

- nonlinear resistance and reactance
- intermodulation of two microwave tones
- harmonic generation
- power-dependence of insertion loss, resonant frequency, Q

Microwave Electrodynamics of Superconductors

- **Why are Superconductors so Useful at Microwave Frequencies?**
- **The Two-Fluid Model**
- **London Equations**
- **BCS Electrodynamics**
- **Nonlinear Surface Impedance**

Why are Superconductors so Useful at Microwave Frequencies?

Low Losses:

Filters have low insertion loss ➔ Better S/N, can be made small

NMR/MRI SC RF pickup coils ➔ x10 improvement in speed

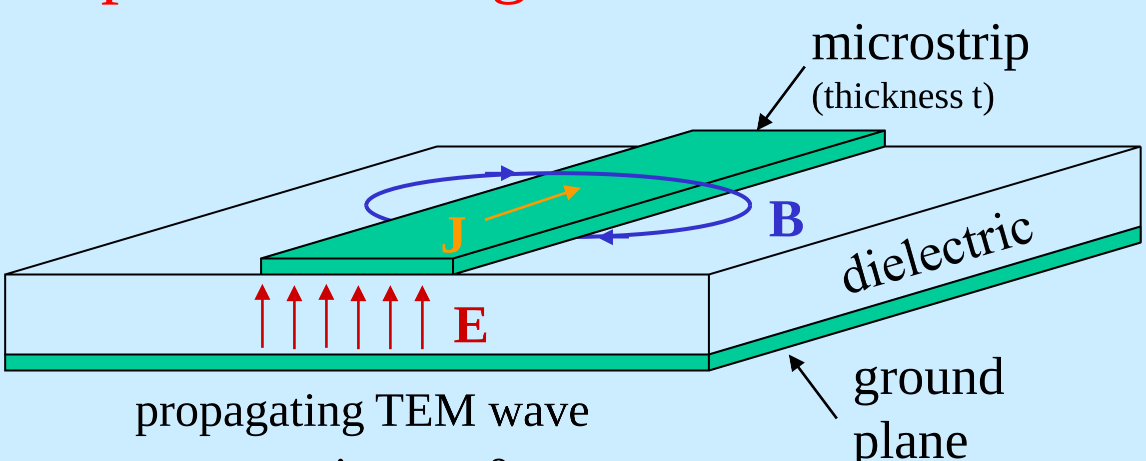
High Q ➔ Steep skirts, good out-of-band rejection

Low Dispersion:

SC transmission lines can carry short pulses with little distortion

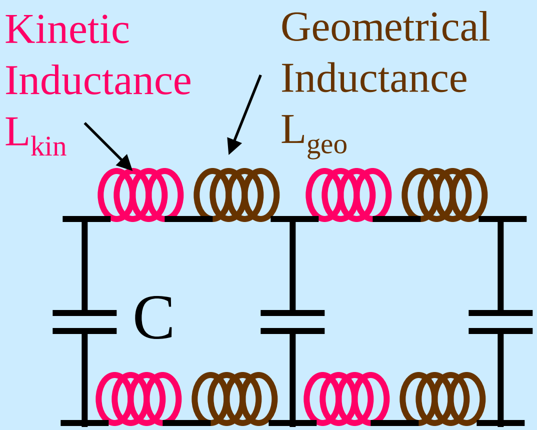
RSFQ logic pulses – 1 ps long, ~2 mV in amplitude: $\int V(t)\,dt = \Phi_0 = 2.07\ \mathrm{mV} \cdot \mathrm{ps}$

Superconducting Transmission Lines



propagating TEM wave

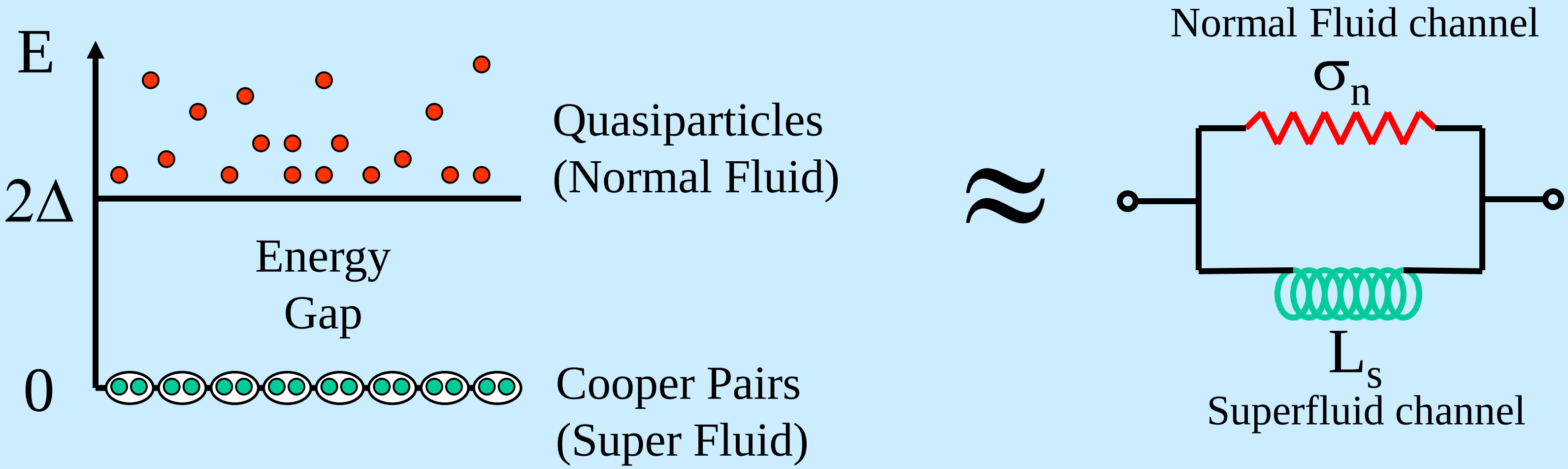
attenuation α ~ 0



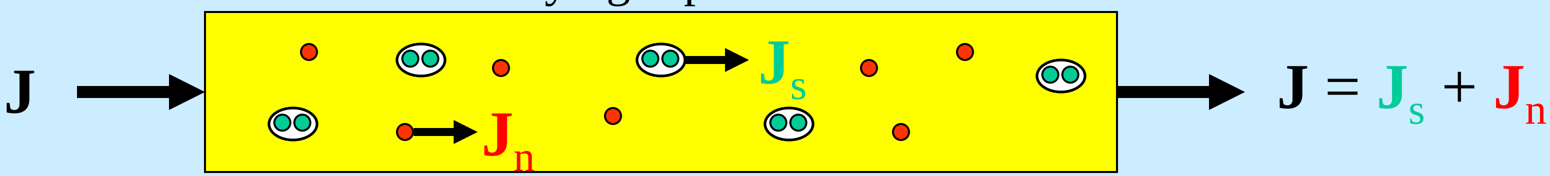
$$L_{kin} \sim \frac{\lambda^2}{t} \qquad v_{phase} = \frac{1}{\sqrt{LC}}$$

$L = L_{kin} + L_{geo}$ is frequency independent

Electrodynamics of Superconductors In the Meissner State



Current-carrying superconductor

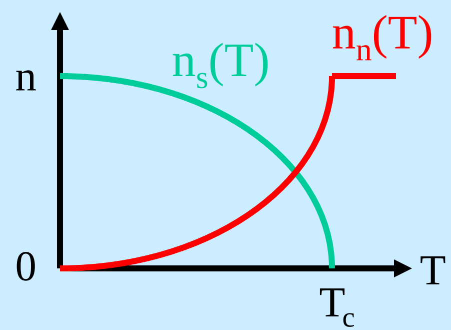


$$J = \sigma E$$

$$\sigma = \sigma_n - i\,\sigma_2$$

$$\sigma_n = n_n e^2 \tau / m$$

$$\sigma_2 = n_s e^2 / m\omega$$



n_n = number of QPs
n_s = number of SC electrons
τ = QP momentum relaxation time
m = carrier mass
ω = frequency

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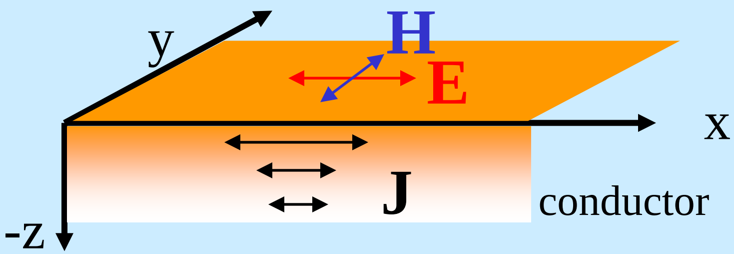
Surface Impedance

$$Z_s = R_s + iX_s = \frac{\left|\vec{E}_{\parallel}\right|}{\int \vec{J}_{\parallel}(z)\,dz} = \sqrt{\frac{i\omega\mu}{\sigma}}$$



Surface Resistance R_s: Measure of Ohmic power dissipation

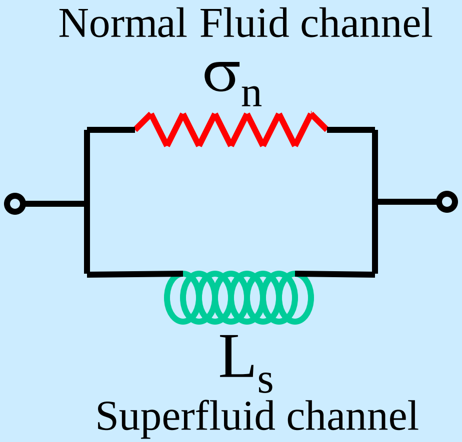
$$P_{Dissipated} = \frac{1}{2}\mathrm{Re}\left\{\iiint_{Volume} \vec{J}\cdot\vec{E}\,dV\right\} = \frac{1}{2}R_s \iint_{Surface}\left|\vec{H}\right|^2 dA \sim \frac{1}{2}I^2 R_s$$

Surface Reactance X_s: Measure of stored energy per period

$$W_{Stored} = \frac{1}{2}\iiint_{Volume}\left(\mu\left|\vec{H}\right|^2 + \mathrm{Im}\left\{\vec{J}\cdot\vec{E}\right\}\right)dV = \frac{1}{2\omega}X_s \iint_{Surface}\left|\vec{H}\right|^2 dA \sim \frac{1}{2}LI^2$$

$$X_s = \omega L_s = \omega\mu\lambda$$

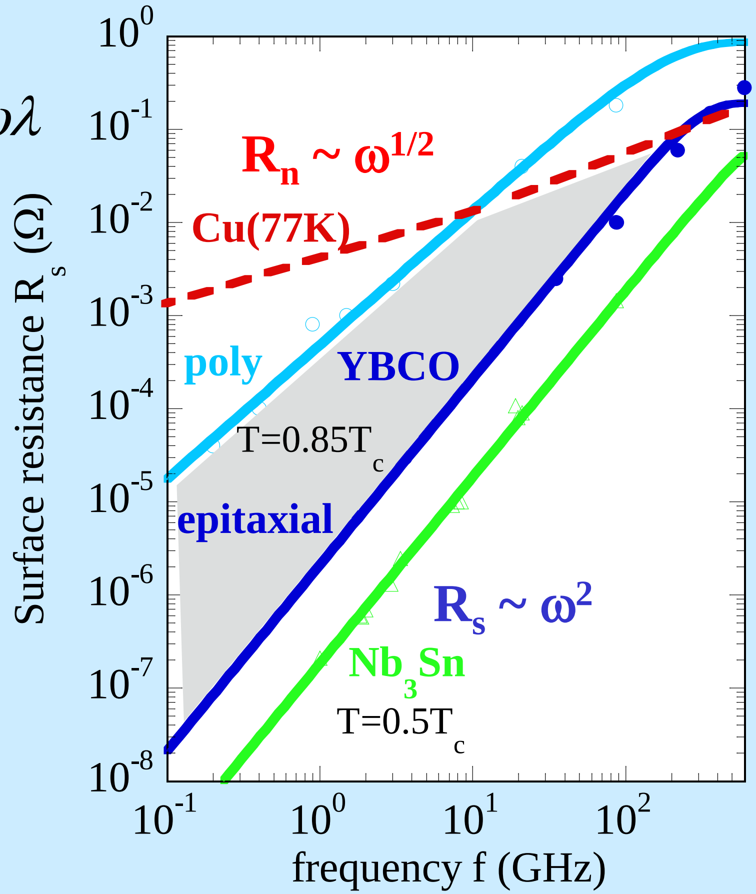
Two-Fluid Surface Impedance



$$Z_s = R_s + iX_s$$

$$R_s = \frac{1}{2}\omega^2 \mu_0 \lambda^3 \sigma_n$$

$$X_s = \mu_0 \omega \lambda$$



M. Hein, Wuppertal

Because $R_s \sim \omega^2$:
The advantage of HTS over Cu diminishes with increasing frequency

R_s crossover at f ~ 100 GHz at 77 K

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The London Equations

Newton's 2nd Law for a charge carrier

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau}$$

τ = momentum relaxation time
$\mathbf{J}_s = n_s e \mathbf{v}_s$

Superconductor:
$1/\tau \rightarrow 0$

$$\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E} = \frac{1}{\mu_0 \lambda_L^2}\vec{E}$$

1st London Equation

1st London Eq. and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ yield:

$$\frac{d}{dt}\left[\nabla \times \vec{J}_s + \frac{n_s e^2}{m}\vec{B}\right] = 0$$

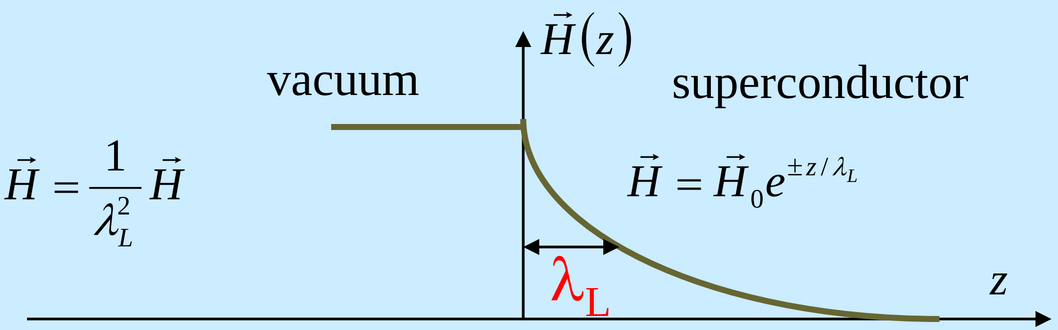
London surmise

$$\nabla \times \vec{J}_s + \frac{n_s e^2}{m}\vec{B} = 0$$

2nd London Equation

These equations yield the Meissner screening

$$\nabla^2 \vec{H} = \frac{1}{\lambda_L^2}\vec{H}$$



$$\lambda_L \equiv \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

λ_L is frequency independent ($\omega < 2\Delta$)

$\lambda_L \sim 20 - 200$ nm

The London Equations continued

	Normal metal	Superconductor	
E is the source of J_n	$\vec{J}_n = \sigma_n \vec{E}$	$\frac{d\vec{J}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} \vec{E}$	**E**=0: J_s goes on forever
Lenz's Law	$\frac{d}{dt}\left[\nabla \times \vec{J}_n + \frac{1}{\mu_0 \lambda_L^2} \vec{B}\right] = 0$	$\mu_0 \lambda_L^2 \left(\nabla \times \vec{J}_s\right) = -\vec{B}$	**B** is the source of J_s, spontaneous flux exclusion

1st London Equation ➔ **E** is required to maintain an ac current in a SC

Cooper pair has finite inertia ➔ QPs are accelerated and dissipation occurs

BCS Microwave Electrodynamics

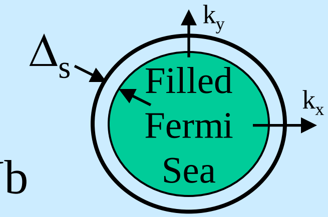
Low Microwave Dissipation

Full energy gap -> R_s can be made arbitrarily small

$$R_s \approx e^{-\Delta(0)/k_B T}$$

for T < T_c/3 in a fully-gapped SC

$$R_s = R_{BCS}(T) + R_{s,\mathrm{residual}}$$



$R_{s,\mathrm{residual}} \sim 10^{-9}\ \Omega$ at 1.5 GHz in Nb

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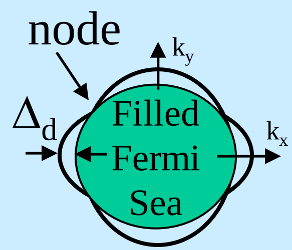
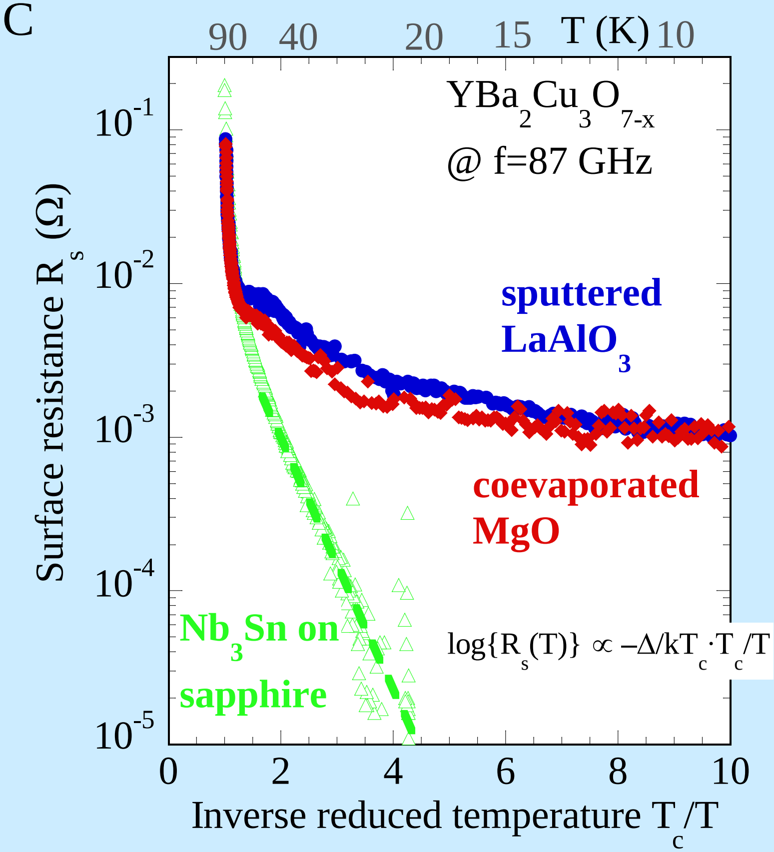
HTS materials have nodes in the energy gap. This leads to power-law behavior of $\lambda(T)$ and $R_s(T)$ and <u>residual losses</u>



$$\lambda(T) = \lambda(0) + a\,T$$

$$R_s = R_{s,\mathrm{residual}} + b\,T$$

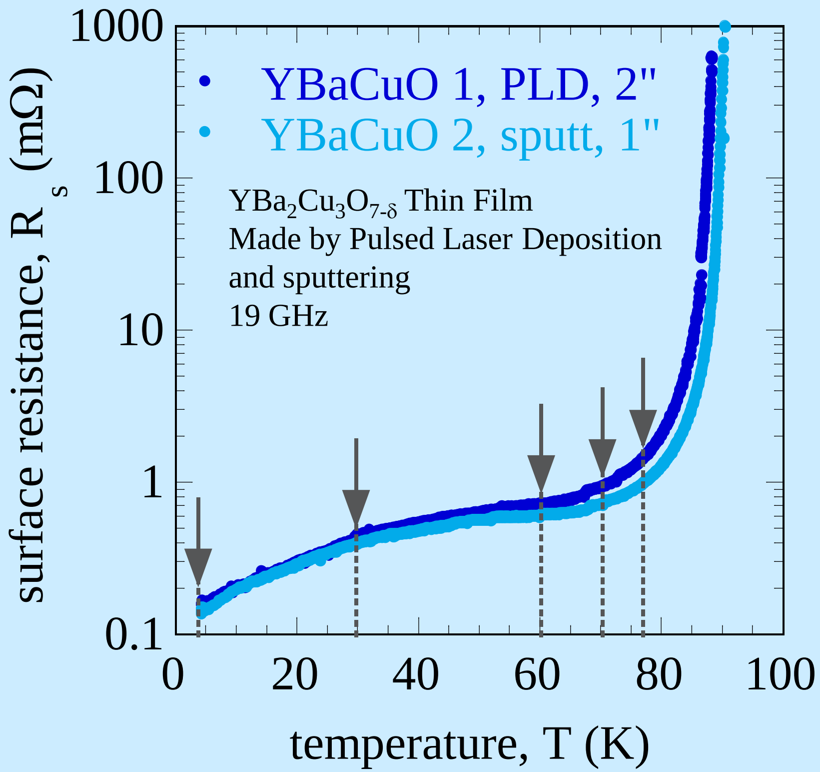
$R_{s,\mathrm{residual}} \sim 10^{-5}\ \Omega$ at 10 GHz in $YBa_2Cu_3O_{7-\delta}$

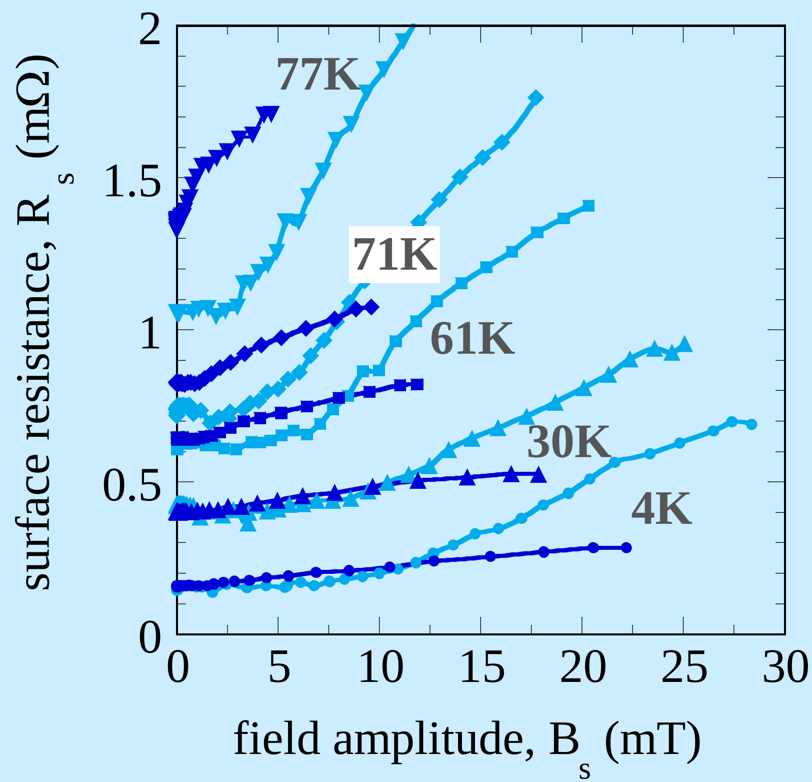


M. Hein, Wuppertal

Nonlinear Surface Impedance of Superconductors

The surface resistance and reactance values depend on the rf current level flowing in the superconductor





Similar results for $X_s(B_s)$

Data from M. Hein, Wuppertal

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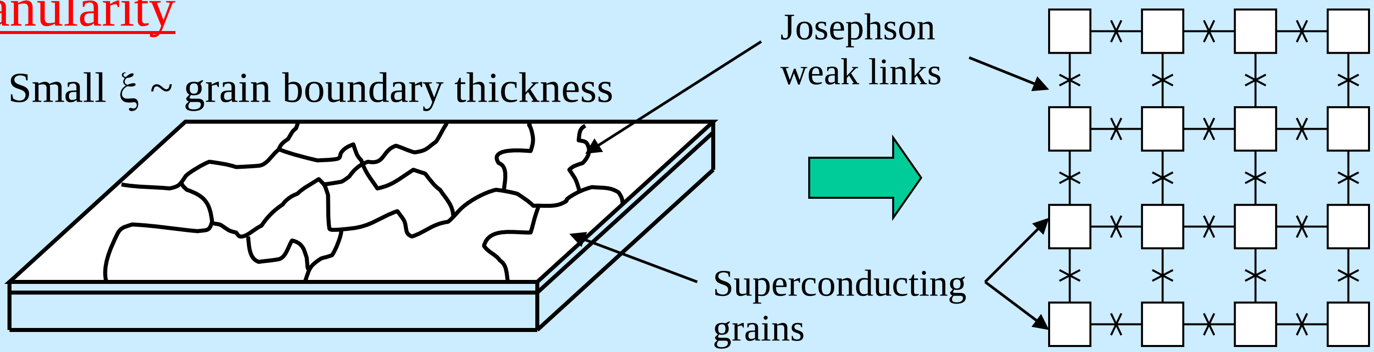
Why are Superconductors so Nonlinear?

Granularity

Small ξ ~ grain boundary thickness



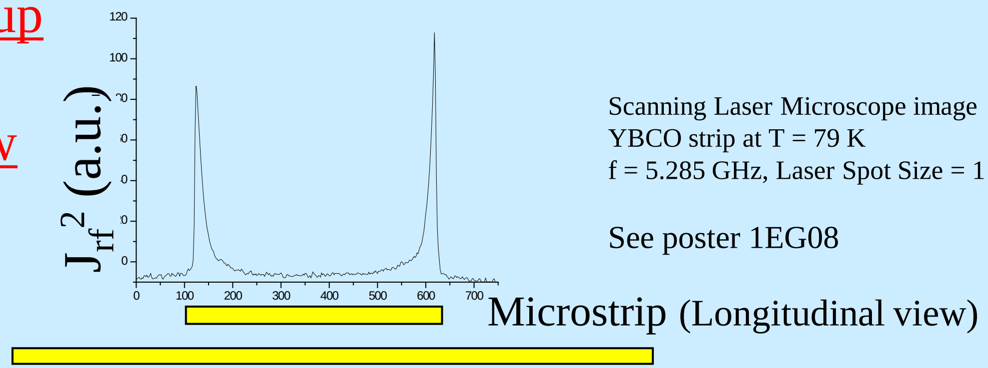
JJs have a strongly nonlinear impedance

McDonald + Clem
PRB 56, 14 723 (1997)

Edge-Current Buildup

+ Vortex Entry and Flow

Heating



Scanning Laser Microscope image
YBCO strip at T = 79 K
f = 5.285 GHz, Laser Spot Size = 1 μm

See poster 1EG08

Intrinsic Nonlinear Meissner Effect

rf currents cause de-pairing – convert superfluid into normal fluid

$$\left(\frac{\lambda(0,T)}{\lambda(J,T)}\right)^2 = 1-\left(\frac{J}{J_{NL}(T)}\right)^2$$

$J_{NL}(T)$ calculated by theory (Dahm+Scalapino)

Nonlinearities are generally strongest near T_c and weaken at lower temperatures

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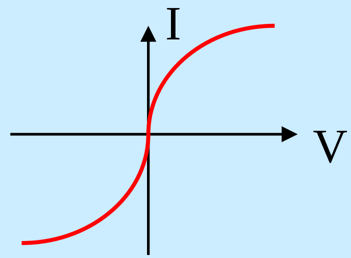
How to Model Superconducting Nonlinearity?

(1) Taylor series expansion of nonlinear I-V curve (Z. Y. Shen)

= 0 if I(-V) = -I(V)

$$I(V) = I(0) + \left(\frac{dI}{dV}\right)_{V=0} \delta V + \frac{1}{2!}\left(\frac{d^2 I}{dV^2}\right)_{V=0} \delta V^2 + \frac{1}{3!}\left(\frac{d^3 I}{dV^3}\right)_{V=0} \delta V^3 + O\left(\delta V^4\right)$$

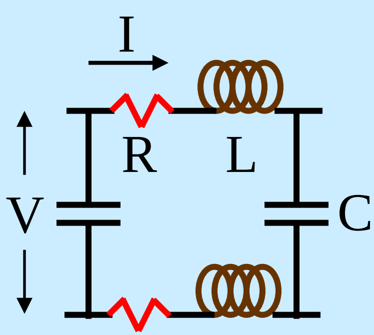


1/R linear term

3rd order term dominates

$V = V_0 \sin(\omega t)$ input yields $\sim V_0^3 \sin(3\omega t) + \ldots$ output

(2) Nonlinear transmission line model (Dahm and Scalapino)



$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} - RI$$

3rd harmonics and 3rd order IMD result

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + RC\frac{\partial I}{\partial t} + C\frac{\partial L}{\partial t}\frac{\partial I}{\partial t} + C\frac{\partial R}{\partial t} I$$

additional terms

L and R are nonlinear:

$$L = L_0 + \Delta L\left(\frac{I}{I_{NL}}\right)^2 \qquad R = R_0 + \Delta R\left(\frac{I}{I_{NL}}\right)^2$$

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Experimental Microwave Superconductivity

- **Cavity Perturbation**
- **Measurements of Nonlinearity**
- **Topics of Current Interest**
- **Microwave Microscopy**

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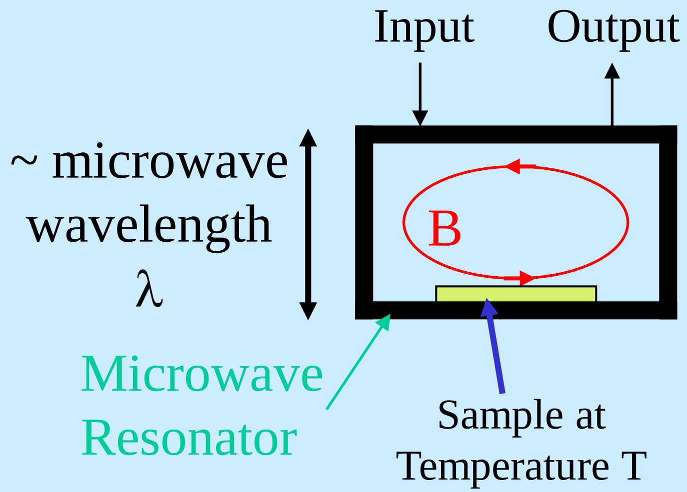
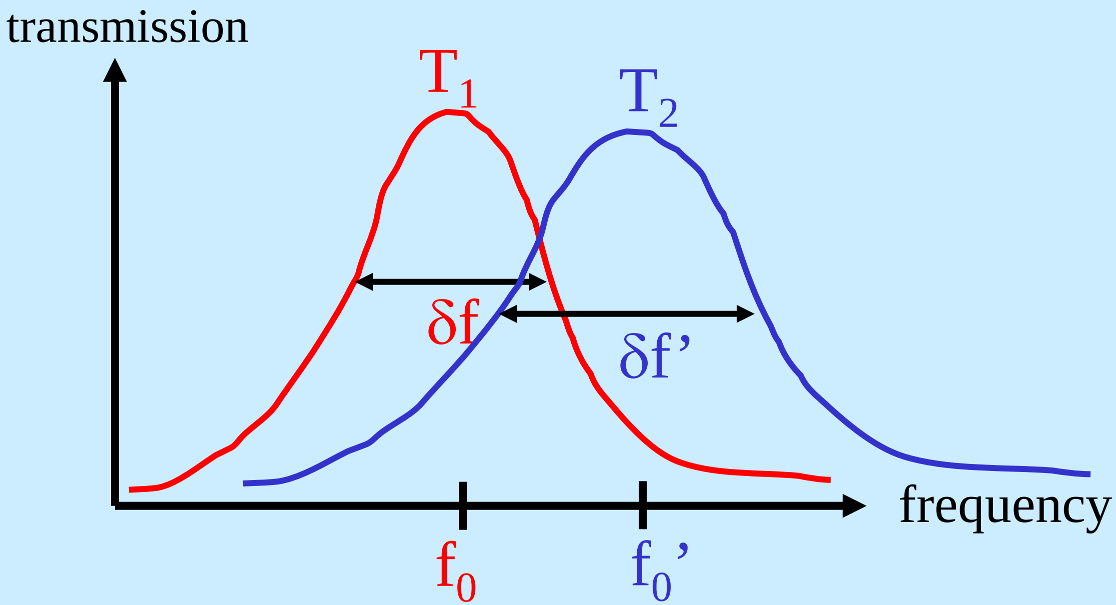
Cavity Perturbation

Objective: determine R_s, X_s (or σ_1, σ_2) from f_0 and Q measurements of a resonant cavity containing the sample of interest





Quality Factor

$$Q = \frac{E_{\text{Stored}}}{E_{\text{Dissipated}}} = \frac{f_0}{\delta f}$$

$\Delta f = f_0' - f_0 \propto \Delta(\text{Stored Energy})$

$\Delta(1/2Q) \propto \Delta(\text{Dissipated Energy})$

Cavity perturbation means $\Delta f << f_0$

$$R_s = \frac{\Gamma}{Q} \qquad \Delta X_s = \frac{2\Gamma}{\omega}\Delta\omega$$

Γ is the sample/cavity geometry factor

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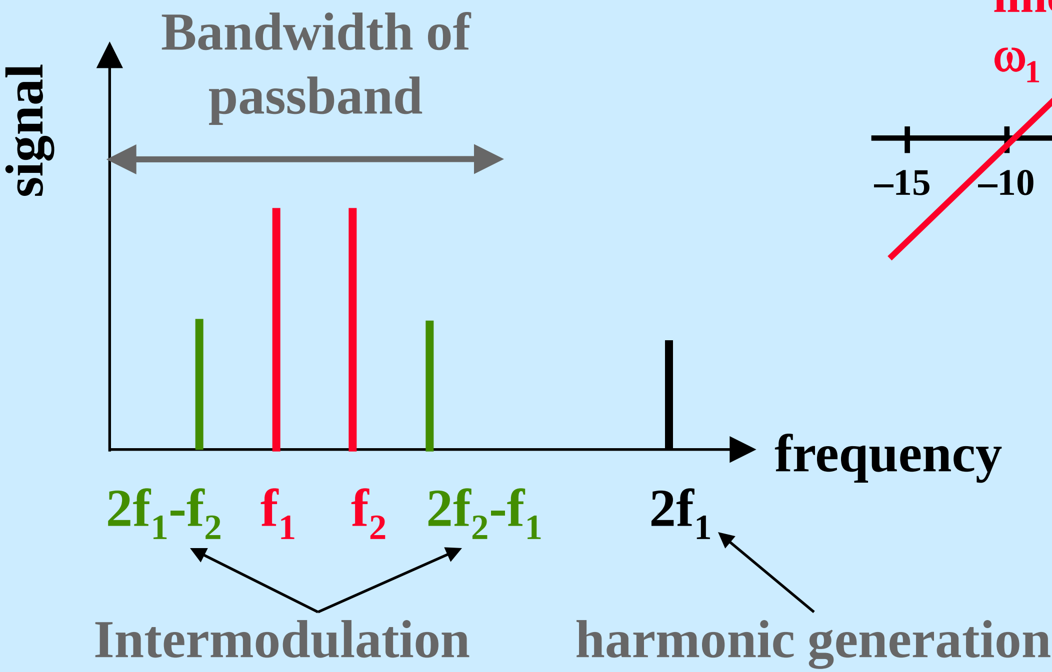
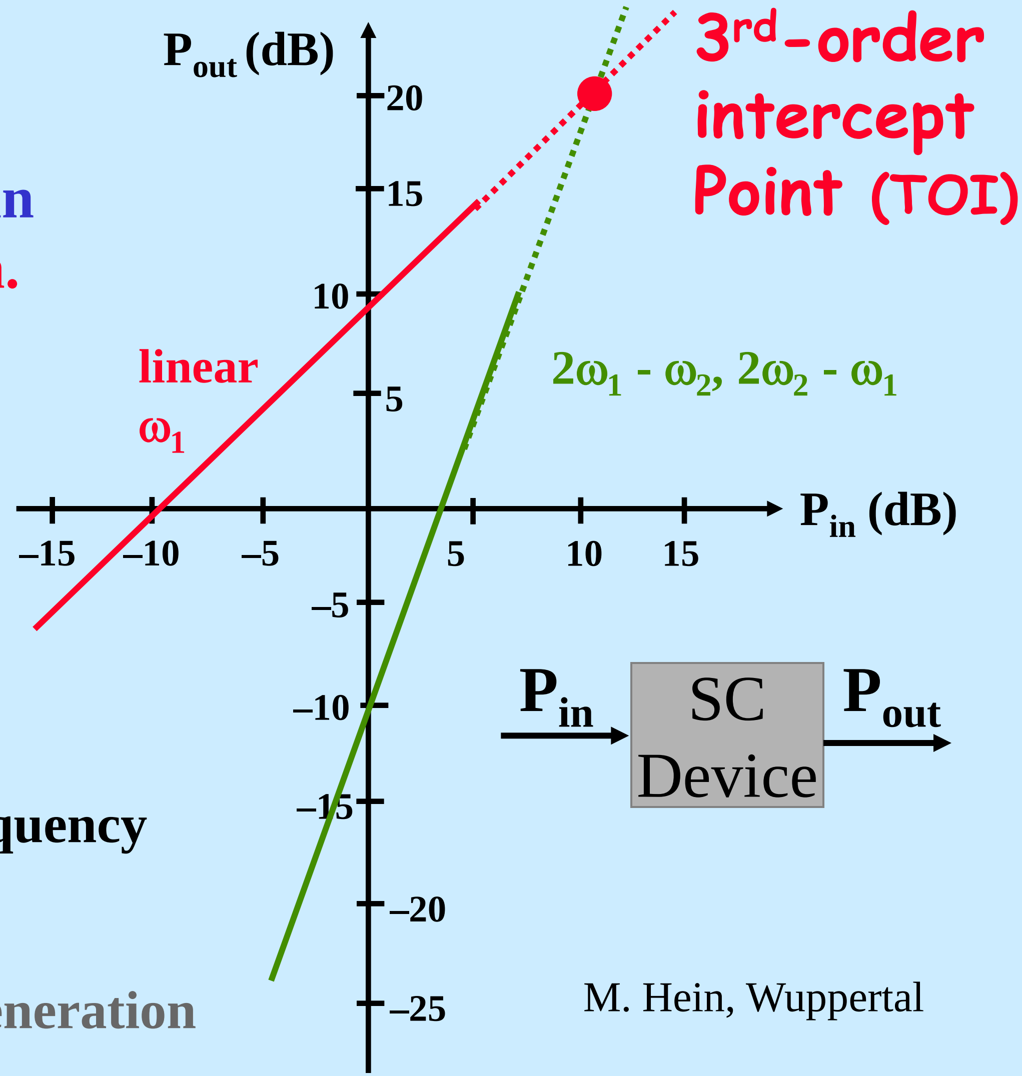
Measurement of Nonlinearities

Intermodulation is a practical problem

Nonlinear (i. e., signal strength dependent) microwave response induces undesirable signals within the passband by intermodulation.





M. Hein, Wuppertal

Topics of Current Interest In Microwave Superconductivity Research

Identifying and eliminating the microscopic sources of extrinsic nonlinearity

- Increase device yield
- Allows further miniaturization of devices
- Will permit more transmit applications

Identify the additional Drude term now seen in $\sigma(\omega,T)$

- under-doped cuprates show $\sigma_2>0$ above T_c
- pseudo-gap electrodynamics

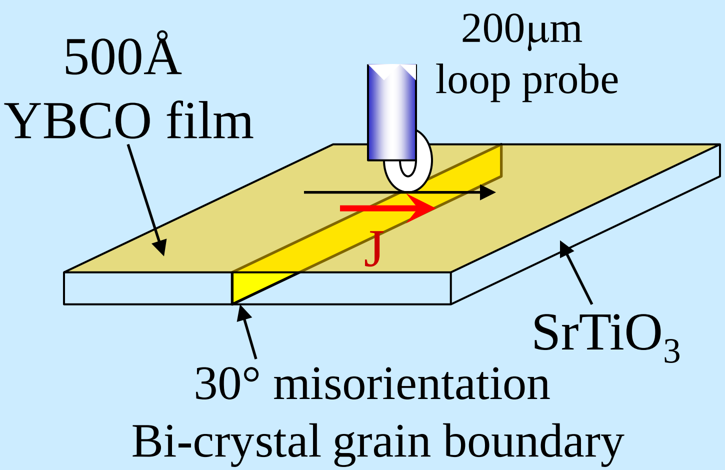
Nonlinear and Tunable Dielectrics

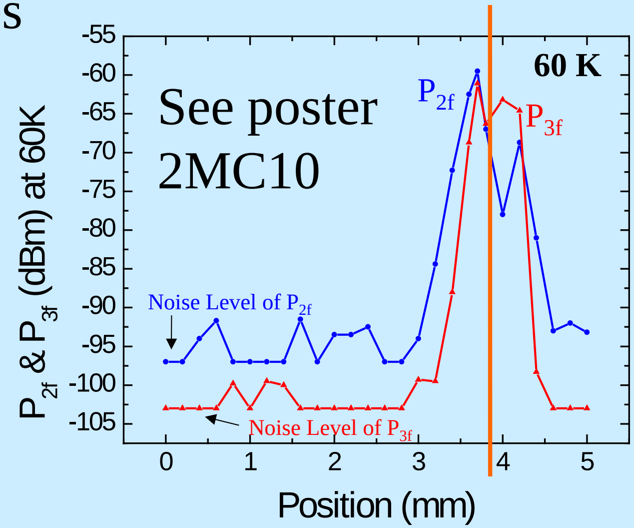
- MgO substrates have a nonlinear dielectric loss at low temperatures
- Ferroelectric and incipient ferroelectric materials as tunable microwave dielectrics/capacitors

Microwave Microscopy of Superconductors

Use near-field optics techniques to obtain super-resolution images of:

1) Materials Properties: Nonlinear response
2) RF fields in operating devices

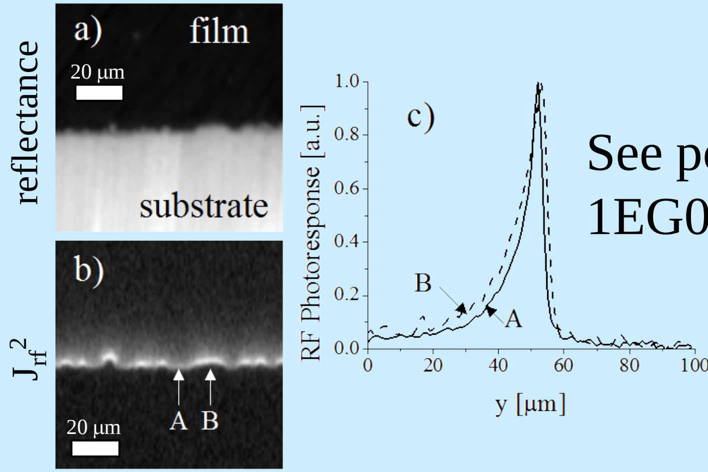




Scanning Laser Microscopy

Image $J_{rf}^2(x,y)$ in an operating superconducting microwave device

Image J_{IMD}



See poster 1EG08

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