Experimental Investigation of Universal Fluctuations in Quantum/Wave Chaotic Scattering Systems

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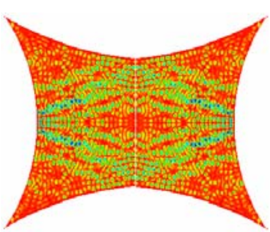
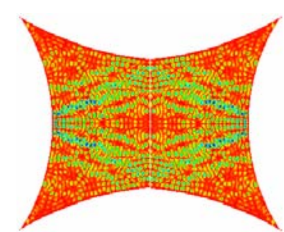
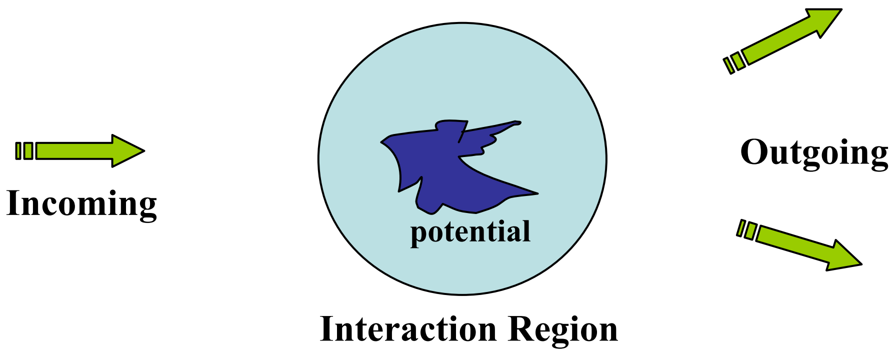


Project funded by the USAF-MURI and DURIP programs



Quantum / Wave Scattering Systems



Described in terms of:
Scattering (S) matrix
Reaction (iK) matrix
Impedance (Z) matrix
Admittance (Y) matrix
…

Complicated function of energy, potential strength, direction, configuration, etc.

Examples: Nuclear scattering
Transport through 2D quantum dots
Electromagnetic scattering
Acoustic scattering
…

What general (detail-independent) properties of these systems can be deduced?

Outline

- Quantum Chaos in Closed Systems
- The Random Matrix Hypothesis
- Our Microwave Cavity / Quantum Dot Experiment
- Random Matrix Theory and Quantum Scattering:
    - The Random Coupling Model (RCM)
- Experimental Tests of the RCM
- Future Work
- Conclusions

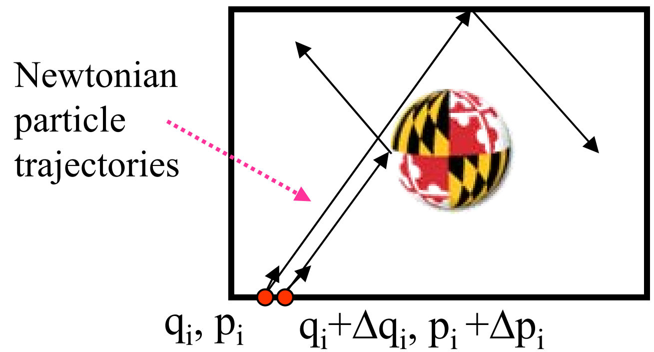
Chaos

Classical: Extreme sensitivity to initial conditions

$$\dot{q}_i = \partial H / \partial p_i$$

$$\dot{p}_i = -\partial H / \partial q_i$$

$$H = T + V \quad \text{Hamiltonian}$$



Manifestations of classical chaos:

Chaotic oscillations, difficulty in making long-term predictions, sensitivity to noise, etc.

Time series, iterated maps, Lyapunov exponents, etc.

Wave/Quantum: ???

Heisenberg Uncertainty principle limits knowledge of initial conditions

$\Delta p\, \Delta q > \hbar$

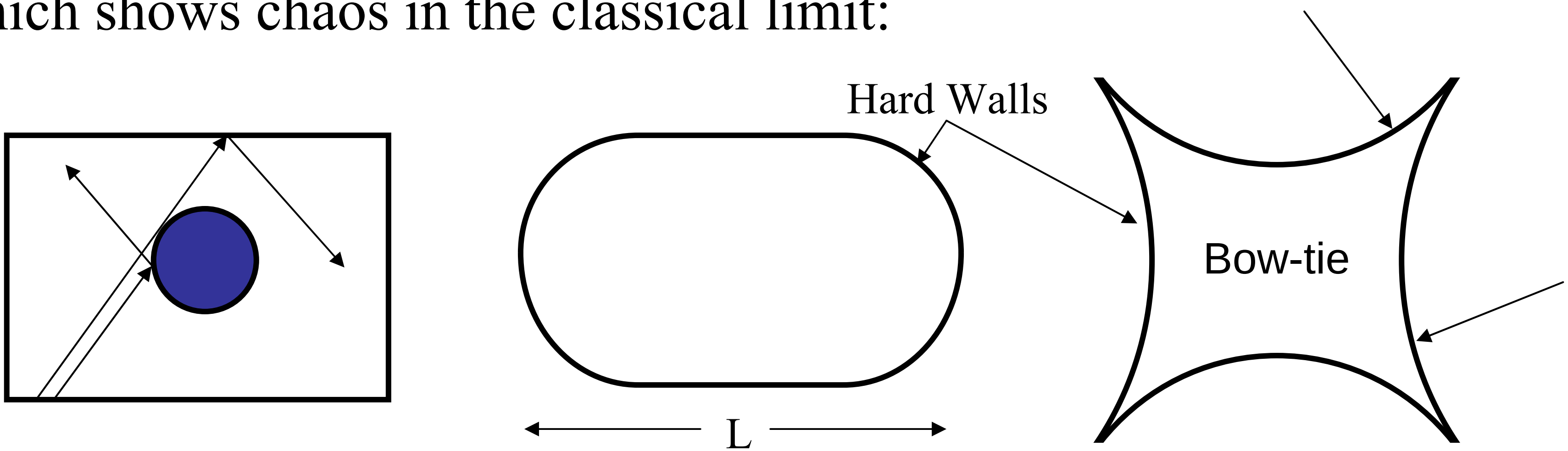
$$\frac{1}{2m}\left(-i\hbar\nabla - qA\right)^2 \Psi + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Manifestations of quantum chaos:

Breaking of degeneracy, Level repulsion, Strong eigenfunction fluctuations, Sensitivity to time-reversal symmetry, etc.

Wave Chaos in Bounded Regions

Consider a two-dimensional infinite square-well potential (i.e. a billiard) which shows chaos in the classical limit:



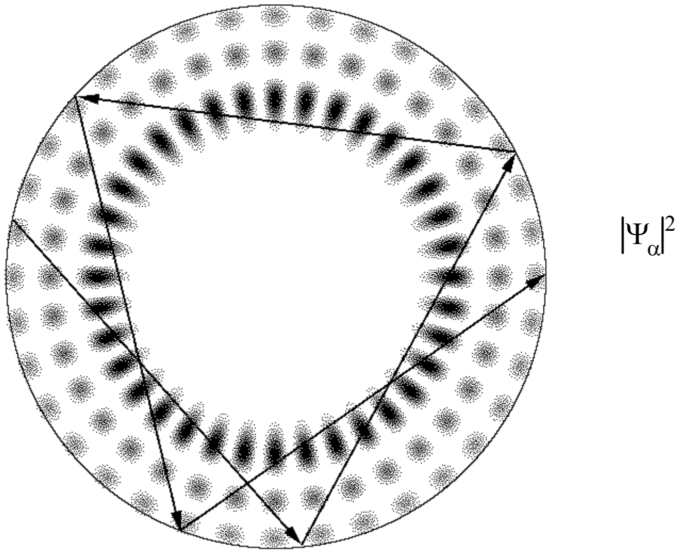
Now solve the electromagnetic wave equation (or Schrodinger equation) in the same potential well

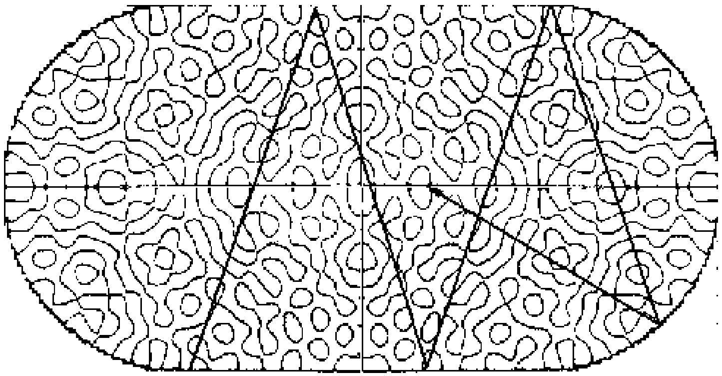
Examine the solutions in the semiclassical regime: $\lambda << L$

What will happen?

Examples

$$\nabla^2 \psi_n + k_n^2 \psi_n = 0$$





<u>Circle:</u>
Trajectories are not chaotic

<u>Stadium:</u>
Trajectories are chaotic

Random Matrix Theory and Quantum Chaos

Hypothesis: Complicated Quantum/Wave systems that have chaotic classical counterparts possess universal statistical properties described by Random Matrix Theory (RMT)

Cassati, 1980
Bohigas, 1984

This hypothesis has been tested in many systems:
Nuclei, atoms, molecules, quantum dots, electromagnetic cavities, acoustics (room, solid body, earth, …), optical resonators, random lasers,…

RMT is an excellent way to quantify your ignorance. It makes no predictions about specific outcomes of experiment or other details.

Some Questions:
Is this hypothesis correct? Is it supported by data?
How far can it be pushed? Can losses be included?
What causes deviations from RMT predictions?

To address these questions, I have to review some more things about chaos and RMT…

Random Matrix Theory (RMT)

Wigner; Dyson; Mehta; …

The RMT Approach:
Complicated Hamiltonian: Nucleus, atom, 2DEG, EM cavity, Acoustic resonator, …

Replace the Hamiltonian (***H***) of these systems by large matrices with randomly distributed elements whose statistical properties are constrained by the symmetry of the system.
Examine the statistical properties of the resulting Hamiltonians

$$H \rightarrow UHU^{-1}$$

U **Orthogonal for Time Reversal Symmetric Systems. (Gaussian Orthogonal Ensemble-GOE) β =1. H real symmetric**

U **Unitary for Time Reversal Symmetry Broken Systems. (Gaussian Unitary Ensemble- GUE) β = 2. H Hermitian**

U **Sympletic for Time Reversal Symmetric and half-integer spin (Gaussian Sympletic Ens. - GSE) β = 4. H self-dual quaternion**

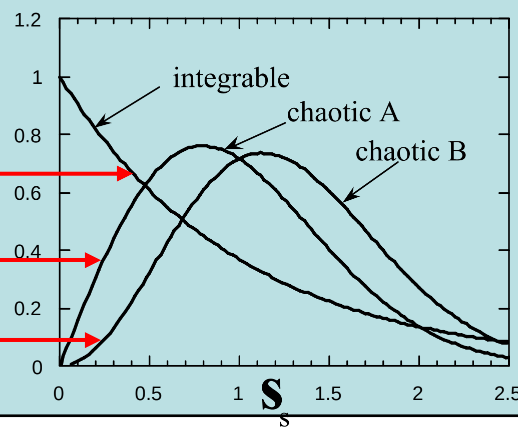
- This simple model makes predictions for **<u>Universal Fluctuations</u>** in the statistical properties of real-world chaotic systems.

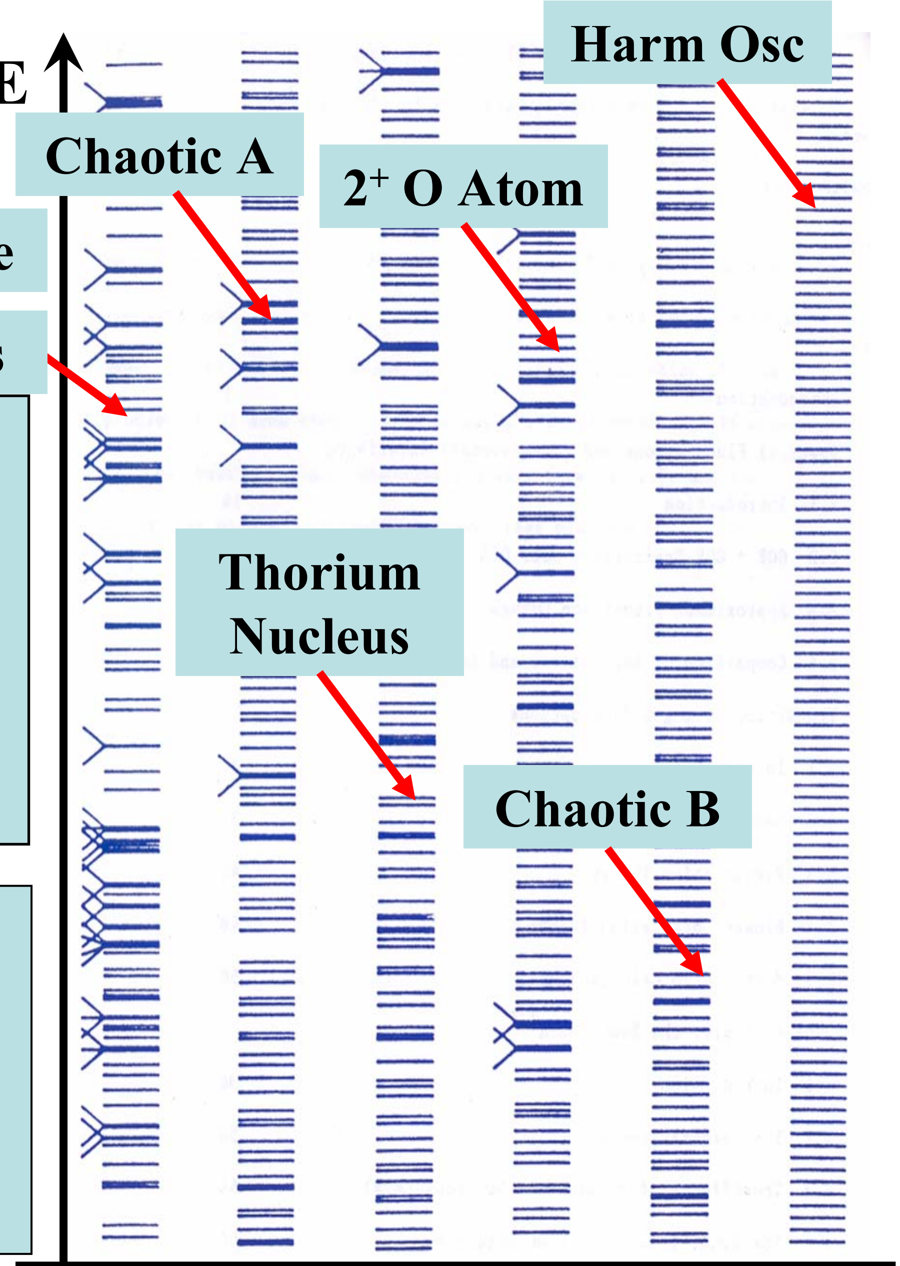
Distribution of Eigen Energies

Normalized Spacing

$$s_n = (E_{n+1} - E_n) / \Delta E$$

Nearest Neighbor Spacing Distributions

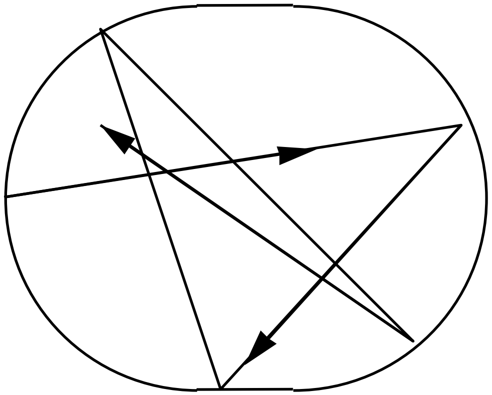




Poisson	$p(s) = Exp(-s)$
TRS (GOE)	$p(s) = \dfrac{\pi}{2} s \cdot Exp(-\dfrac{\pi}{4} s^2)$
TRSB (GUE)	$p(s) = \dfrac{32}{\pi^2} s^2 \cdot Exp(-\dfrac{4}{\pi} s^2)$

GOE→GUE Crossover experiment: P. So, *et al.*, Phys. Rev. Lett. 74 2662 (1994)

Properties of Eigenfunctions



Rays ergodically fill phase space.

Eigenfunctions appear to be a superposition of plane waves with random amplitudes and phases.

Berry Hypothesis

Random Amplitude   Random Direction   Random Phase

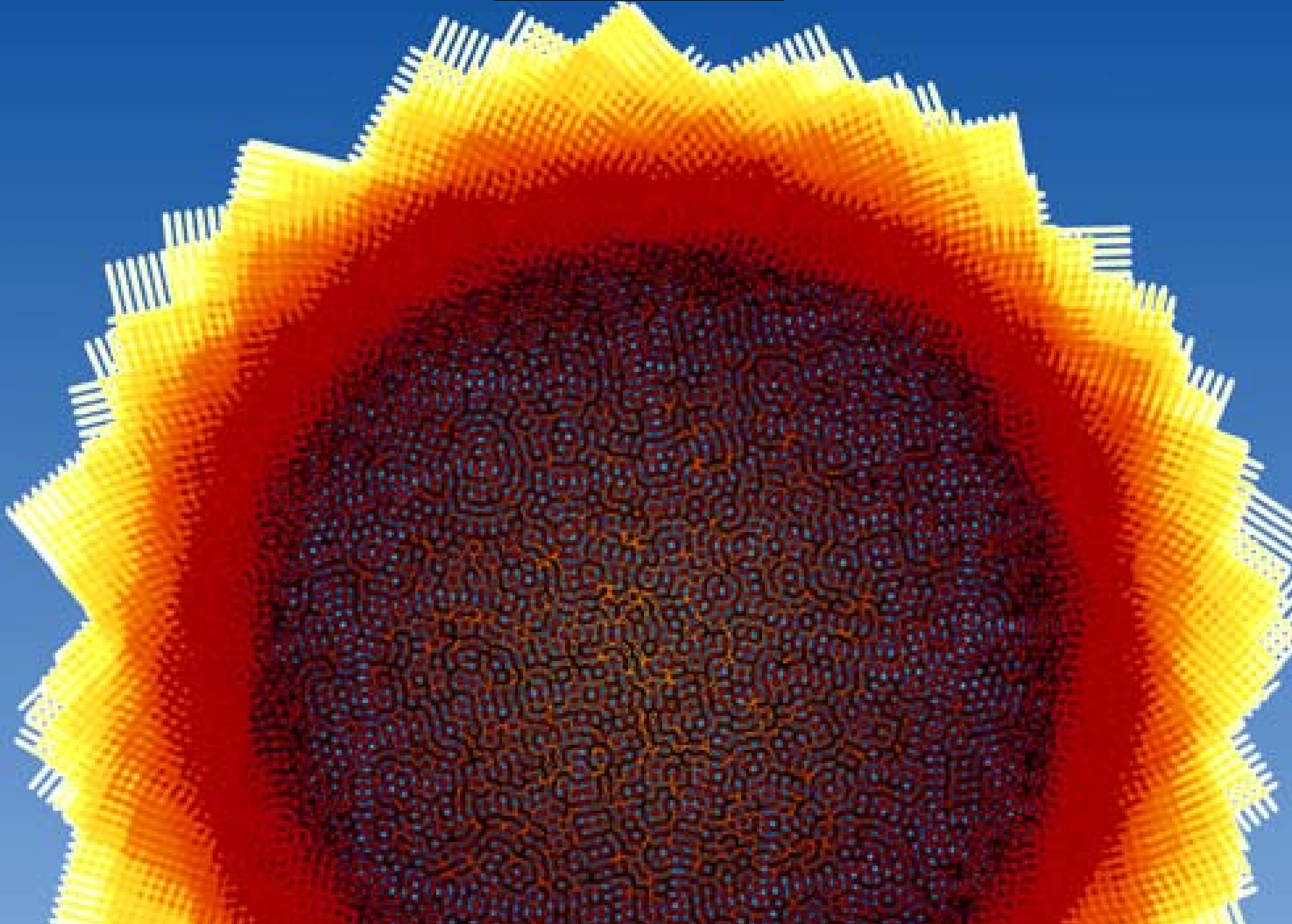
$$\phi_n = Lim_{N\to\infty} \mathrm{Re}\{\sqrt{\frac{2}{AN}}\sum_{j=1}^{N} a_j Exp[j(k_j.x+\theta_j)]\}$$

T.R.S

$\boldsymbol{k}_j$ is uniformly distributed on a circle $|\boldsymbol{k}_j|=\boldsymbol{k}_n$

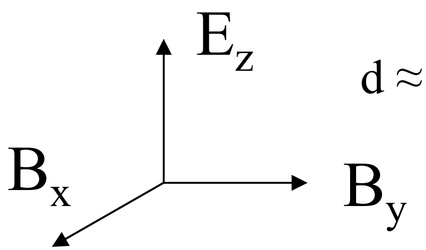
ϕ_n is described by a distribution

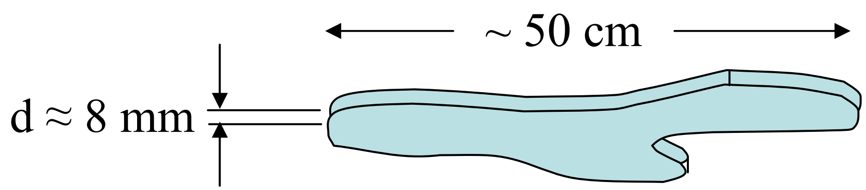
Eric Heller, Harvard



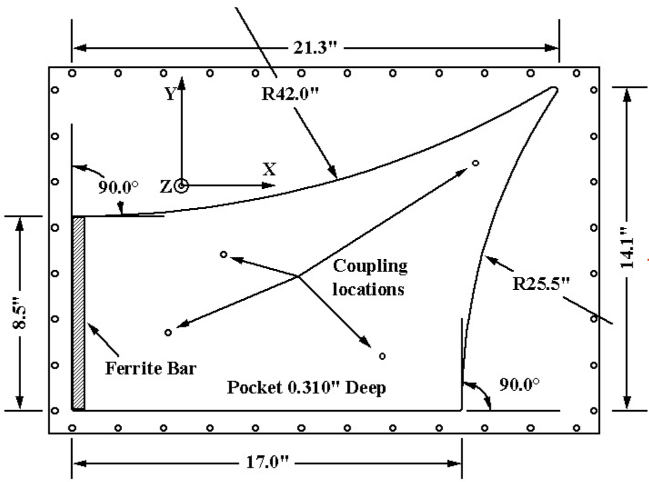
Schrödinger – Helmholtz Analogy

Our Experiment: A clean, zero temperature, quantum dot with no Coulomb or correlation effects! Table-top experiment!





An empty "two-dimensional" electromagnetic resonator



$$\nabla^2 \Psi_n + \frac{2m}{\hbar^2}\left(E_n - V\right)\Psi_n = 0$$

$$\text{with } \Psi_n = 0 \text{ at boundaries}$$

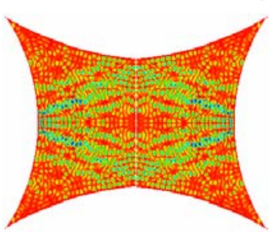
Schrödinger equation

$$\nabla^2 E_{z,n} + k_n^2 E_{z,n} = 0$$

$$\text{with } E_{z,n} = 0 \text{ at boundaries}$$

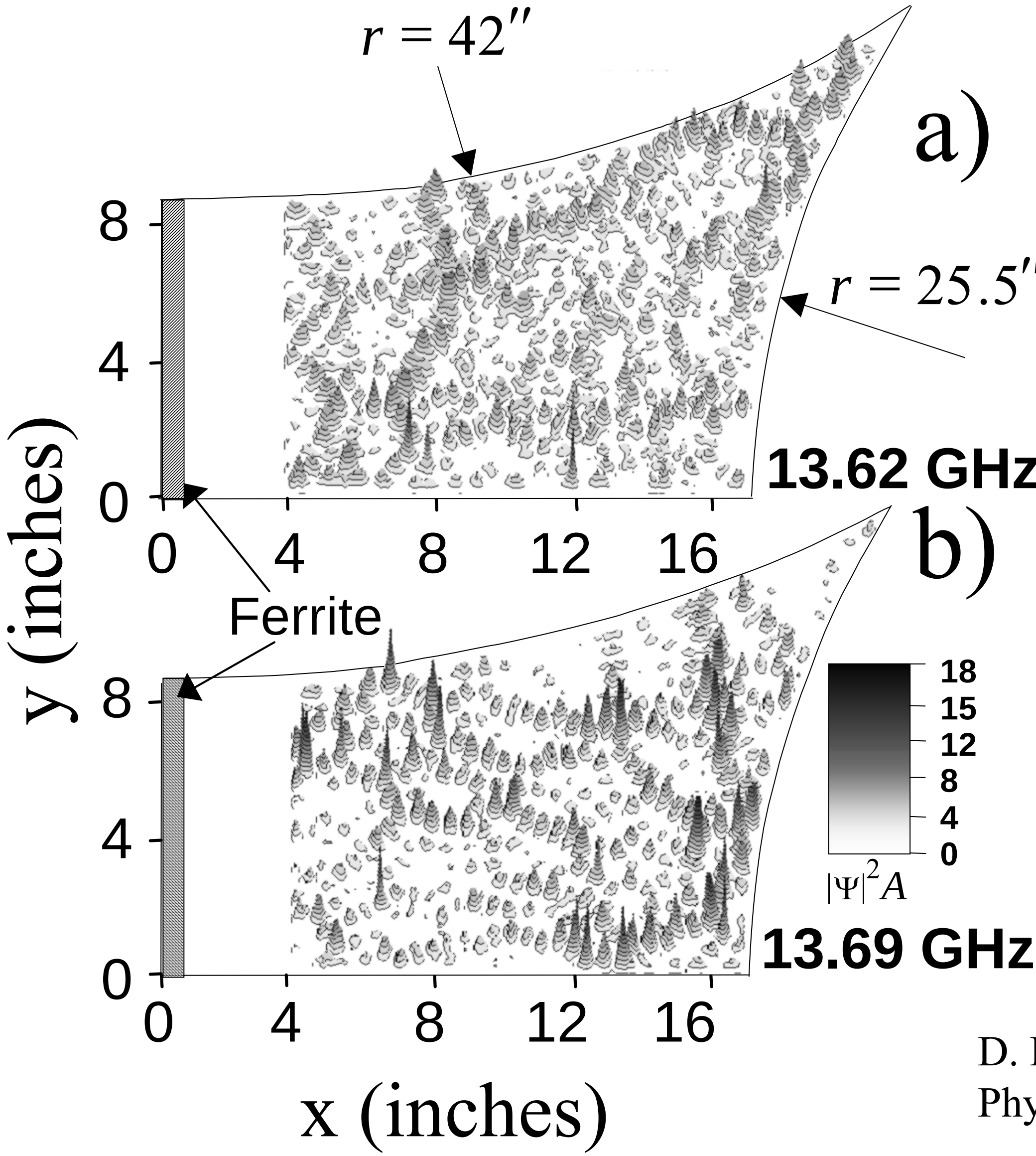
Helmholtz equation

Stöckmann + Stein, 1990



A. Gokirmak, *et al*. Rev. Sci. Instrum. **69**, 3410 (1998).

Wave Chaotic Eigenfunctions with and without Time Reversal Symmetry

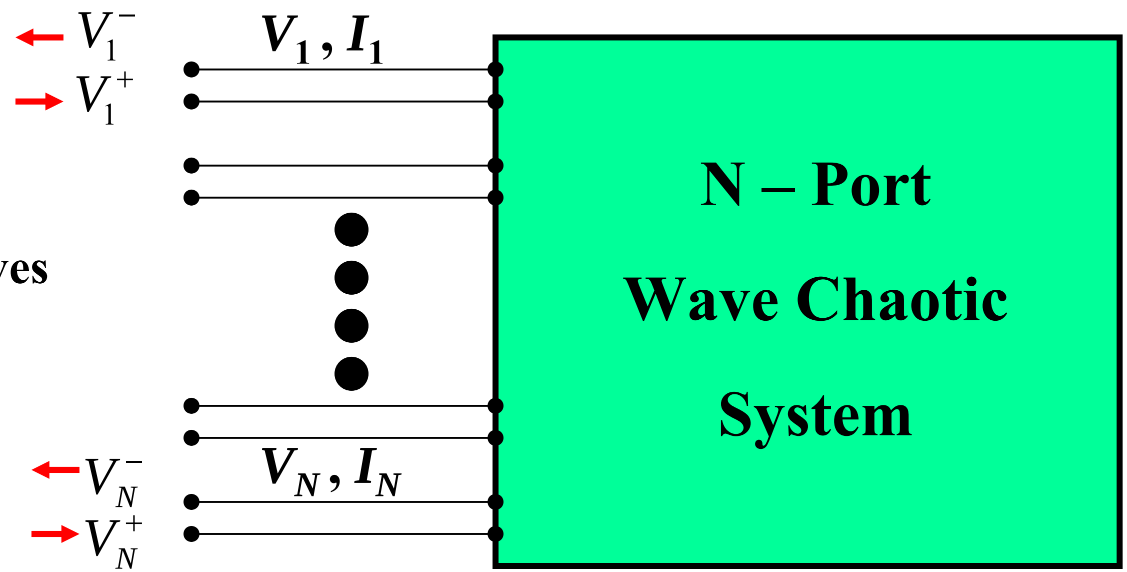


TRS Broken (GUE)

TRS (GOE)

D. H. Wu and S. M. Anlage, Phys. Rev. Lett. 81, 2890 (1998).

Open Systems: Quantum/Wave Chaotic Scattering
N-Port Description of an Arbitrary System



N Ports

- **Voltages and Currents,**
- **Incoming and Outgoing Waves**

S matrix

$$\begin{bmatrix} V^-{}_1 \\ V^-{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^-{}_N \end{bmatrix} = [S] \cdot \begin{bmatrix} V^+{}_1 \\ V^+{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^+{}_N \end{bmatrix}$$

Z matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = [Z] \cdot \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ \bullet \\ I_N \end{bmatrix}$$

Y matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ \bullet \\ I_N \end{bmatrix} = [Y] \cdot \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_N \end{bmatrix}$$

$$Y = Z^{-1}$$

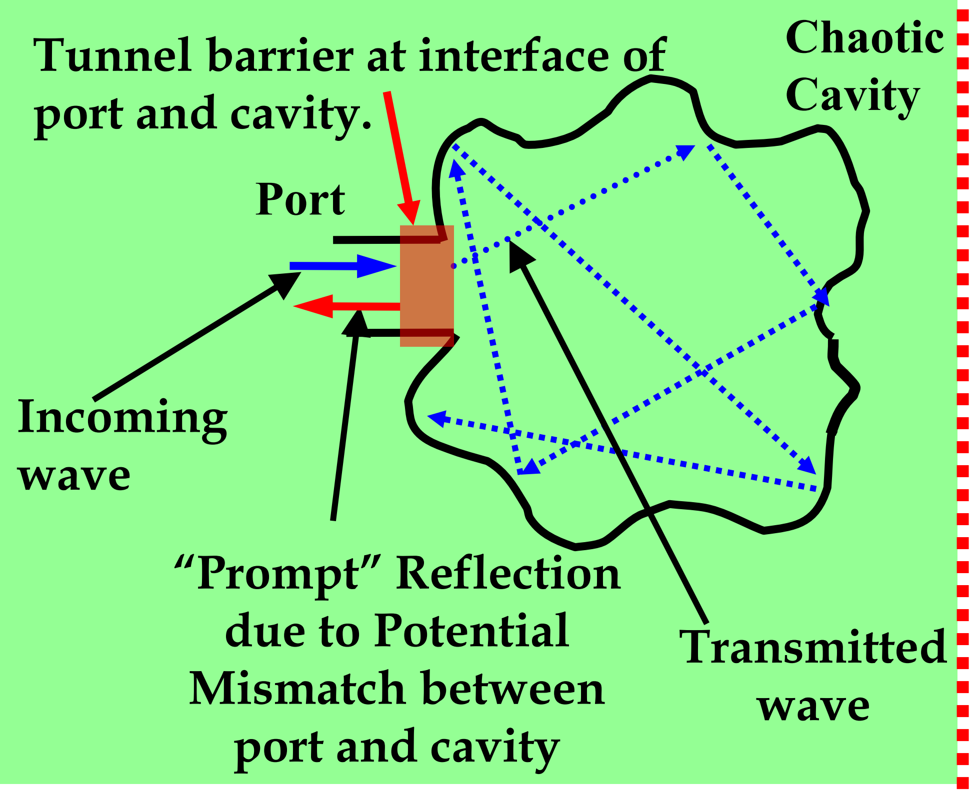
$$S = (Z + Z_0)^{-1}(Z - Z_0)$$

$$Z(\omega), Y(\omega), S(\omega)$$

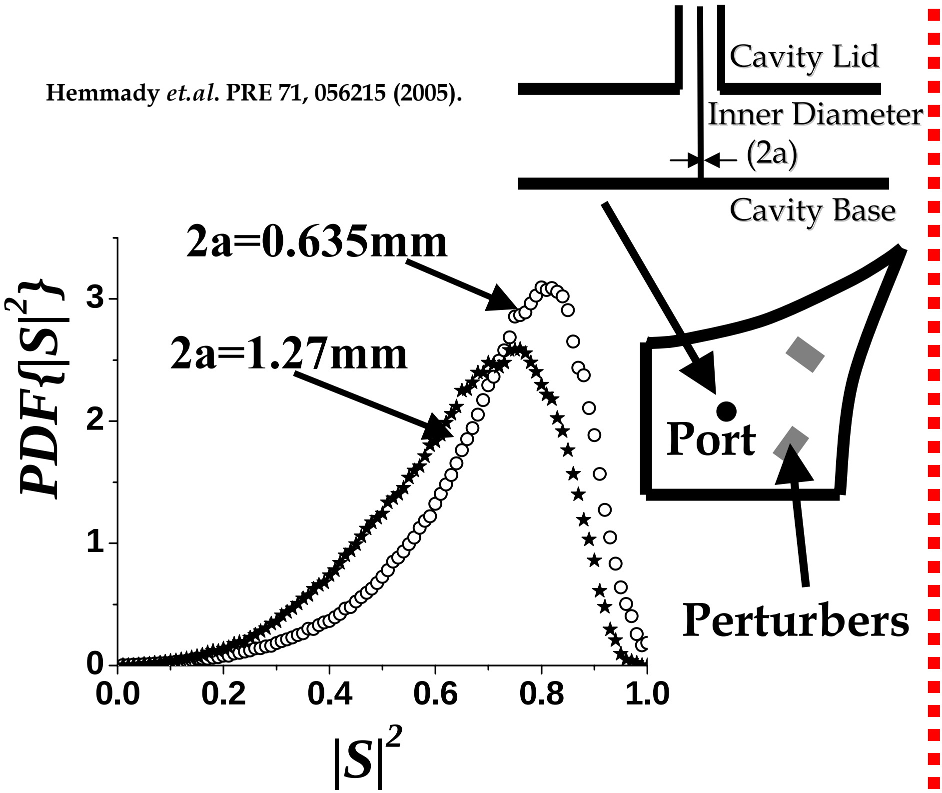
- Complicated Functions of frequency
- Detail Specific (Non-Universal)

The Issue of “Non-Ideal Coupling” :

Almost all theoretical work assumes “Ideal Coupling”

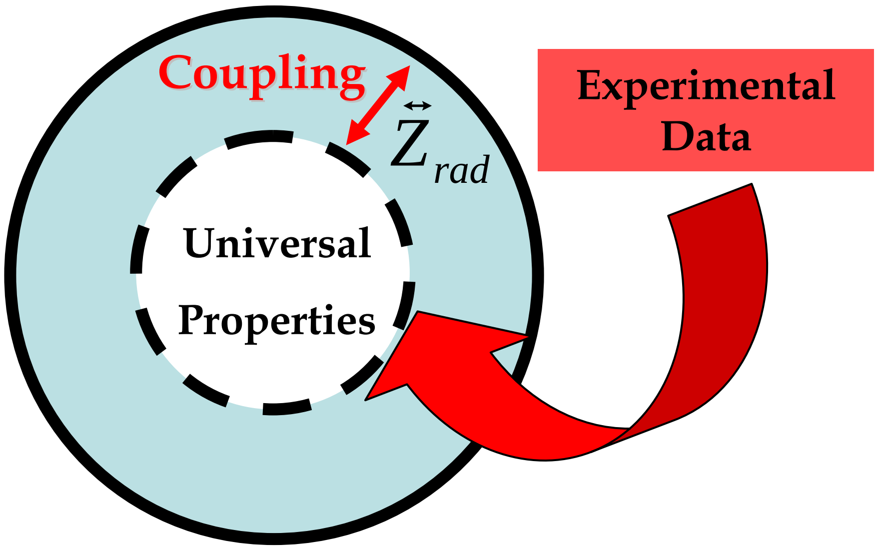


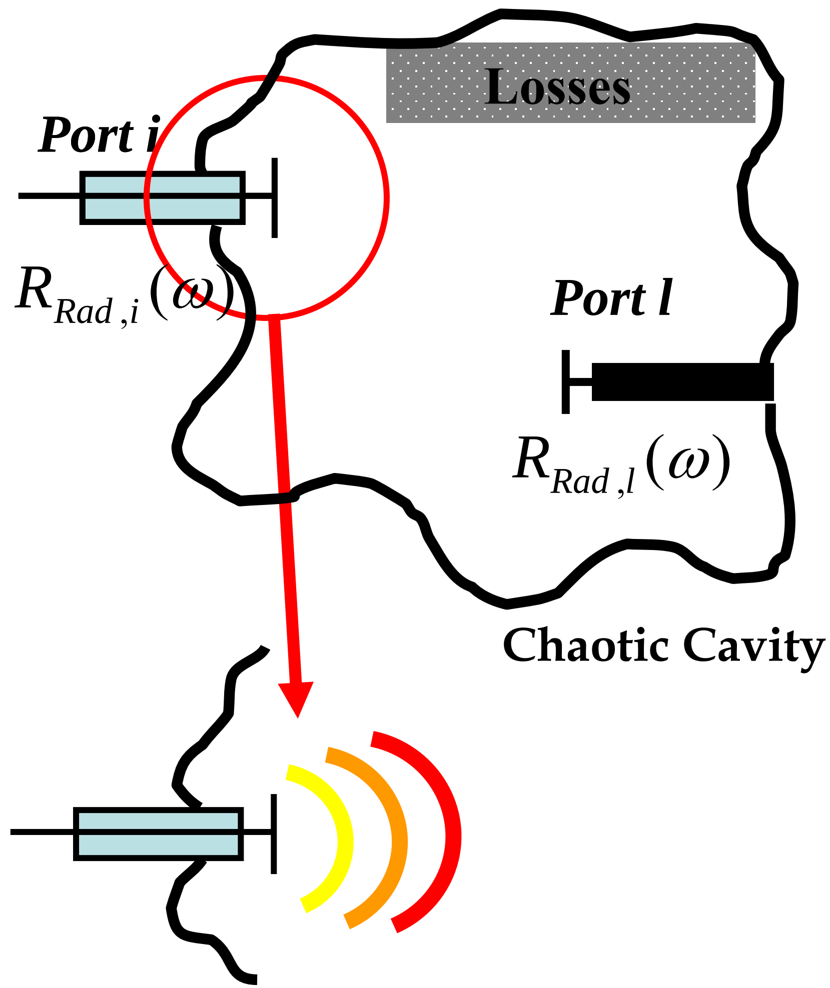
Manifestation of Non-ideal coupling in measured scattering fluctuations.



The Random Coupling Model :

Xing Zheng, Tom Antonsen, Ed Ott





- Coupling is quantified by the measured Radiation Impedance matrix of the ports
- $\overleftrightarrow{Z}_{rad}$ is a deterministic frequency-dependant quantity. Eliminates the need to resort to statistically averaged quantities.

$$Z(k) = -\frac{j}{\pi}\sum_{\text{modes } n} R_{Rad}^{1/2}(k_n)\frac{\Delta k_n^2 \; w_n \; w_n^T}{k^2(1-jQ^{-1})-k_n^2}R_{Rad}^{1/2}(k_n)$$

Free Space Radiation Impedance:

$$Z_{Rad}(\omega) = R_{Rad}(\omega) + jX_{Rad}(\omega)$$

Normalized Cavity Impedance z

When applied to an ensemble of ray-chaotic cavities, the impedance becomes;

$$Z = \underbrace{\overline{Z}}_{\text{Mean part}} + \underbrace{\widetilde{Z}}_{\text{Fluctuating part}} = jX_{Rad} + (\rho + j\xi)R_{Rad}$$

<ρ> = 1
<ξ> = 0

$$z = \rho + j\xi = \frac{Z - jX_{Rad}}{R_{Rad}}$$

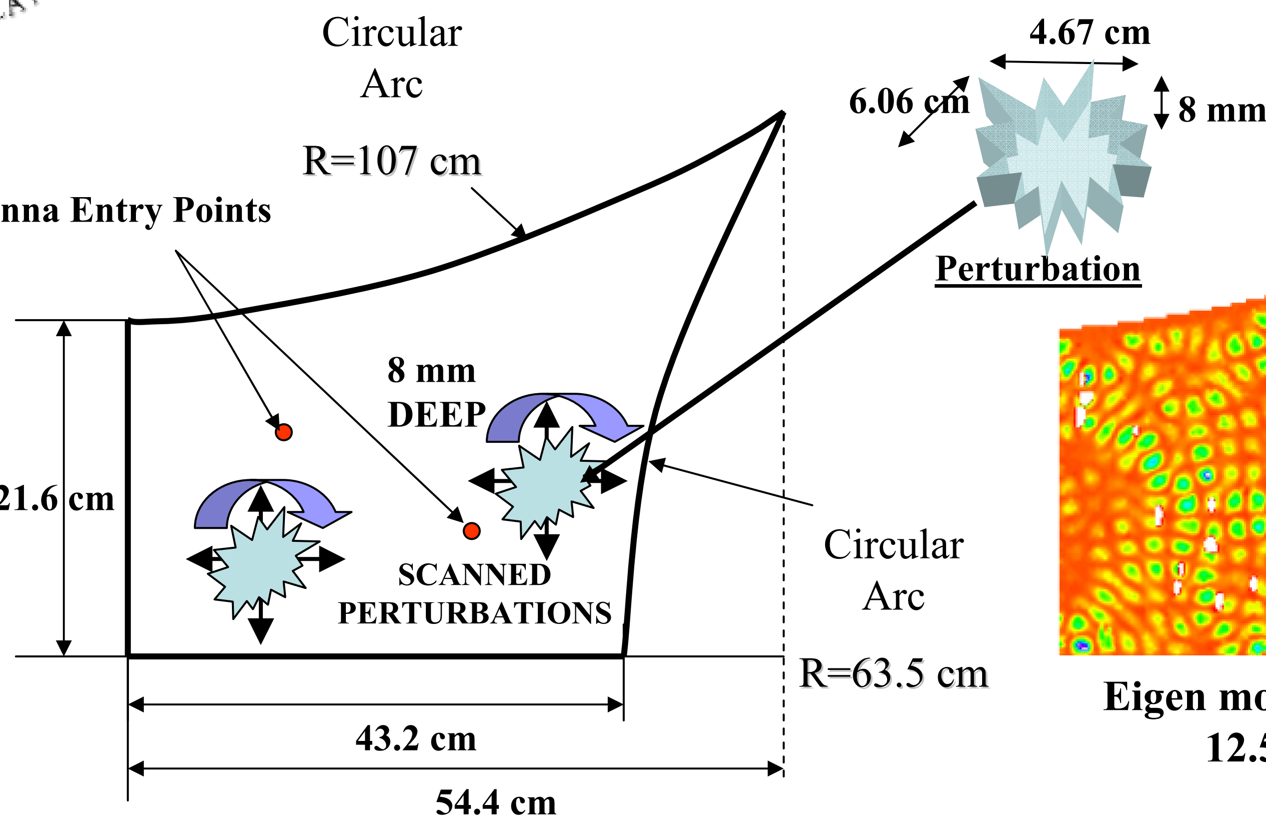
Normalized impedance
"perfect coupled"
ρ, ξ distributions depend only on loss

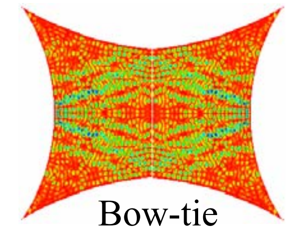
Loss Parameter: $\dfrac{k^2}{\Delta k_n^2 Q} \sim \dfrac{\mathrm{Im}[\omega_0]}{\Delta\omega} \qquad (\sim\gamma)$

With "perfectly coupled" data we can now make direct comparisons with many RMT predictions!

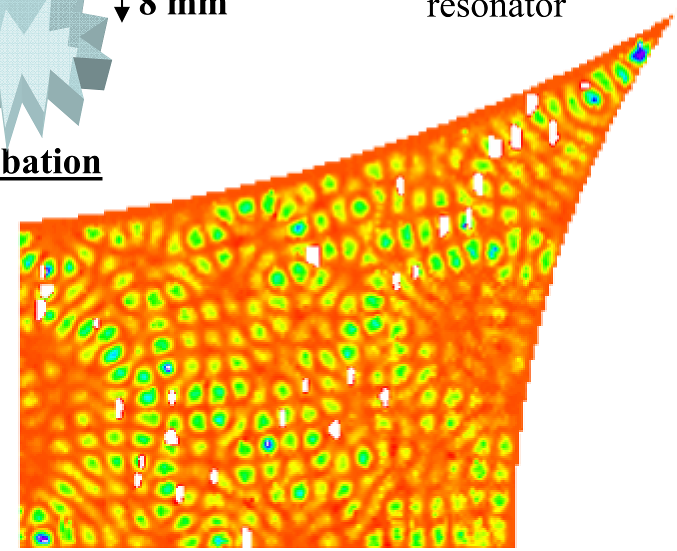
X. Zheng, *et al*. cond-mat/0408317; cond-mat/0408327

¼-Bow-Tie Resonator
EXPERIMENTAL SETUP:





Bow-tie
resonator



Eigen mode Image at
12.57GHz

- 2 Dimensional Quarter Bow Tie Wave Chaotic cavity
- Classical ray trajectories are chaotic - short wavelength - Quantum Chaos
- 1-port, 2-port S and Z measurements in the 3-18 GHz range
- Ensemble average through 100 locations and orientations of the perturbations
- Perturbers are of size ~λ or bigger

What is New About Our Approach?

It allows us to generate a large quantity of data in the "perfectly coupled" limit

All ports have transmission coefficient 1 at all frequencies

Typical measurement:

3 GHz – 18 GHz (~ 800 modes)
16,000 frequency points measured
100 "renditions" in the ensemble

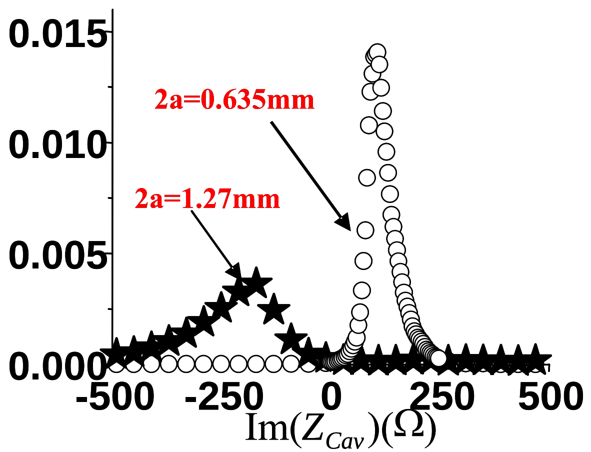
1,600,000 data points in all

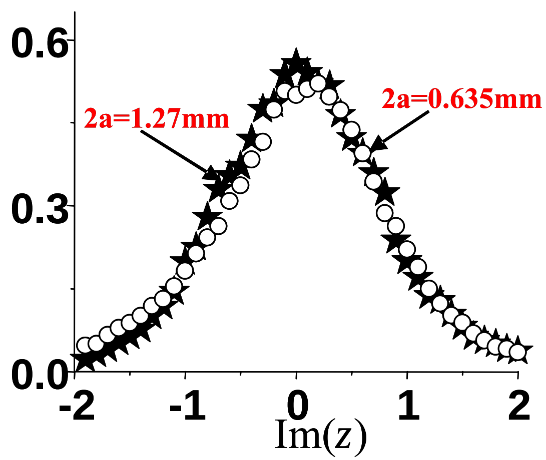
All data is "normalized" to remove the effects of coupling

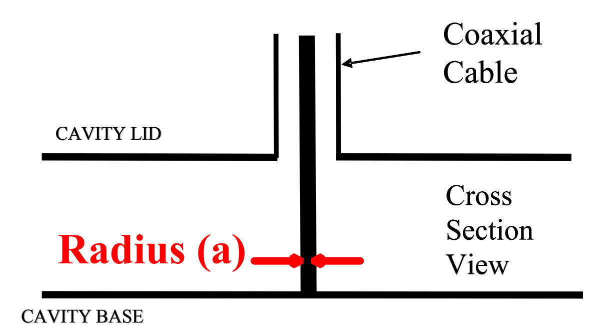
Results: ensembles of complex s, z, y for 1, 2, 3,…-ports

for $\frac{k^2}{\Delta k_n^2 Q} \sim 0.5$ to 30

Subsets of this data are used for statistical analysis







S. Hemmady, *et al.*,
Phys. Rev. Lett. 94, 014102 (2005)

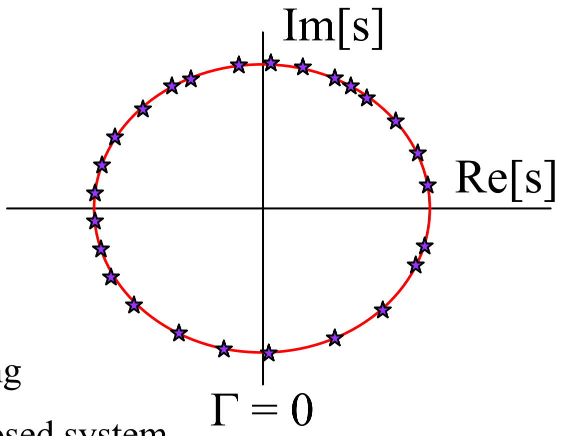
Dyson's circular ensemble hypothesis
How is this affected by loss?

Scattering Matrix: $$S(E) = \frac{1 - iK(E)}{1 + iK(E)}$$

Reaction Matrix: $$K(E) = V^{t} \frac{1}{E + i\frac{\Gamma}{2} - \hat{H}} V$$

Coupling

Hamiltonian of closed system



$\Gamma = 0$: Eigenphases of S uniformly distributed on the unit circle

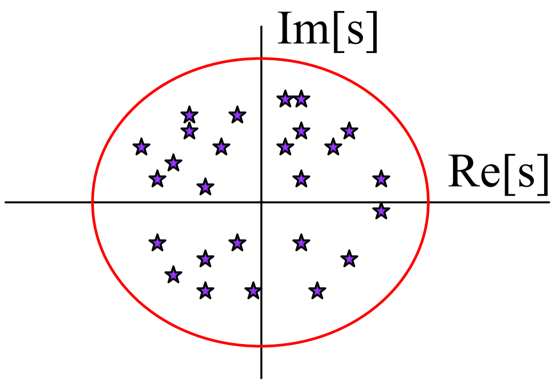
Eigenphase repulsion $$P(\varphi_1, \varphi_1, \ldots \varphi_N) \sim \prod_{n>m} \left| e^{i\varphi_n} - e^{i\varphi_m} \right|^{\beta}$$

$$\beta = \begin{cases} 1 & \text{GOE} \\ 2 & \text{GUE} \\ 4 & \text{GSE} \end{cases}$$

$\Gamma \neq 0$: No detailed predictions

$|s|$ and ϕ_s should be statistically independent

ϕ_s correlations ???



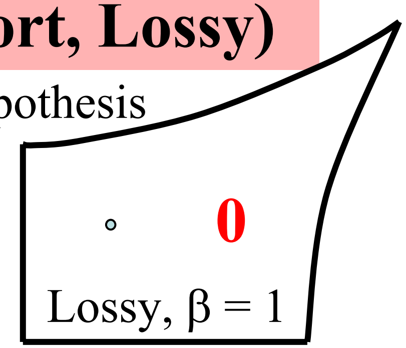
Statistical Independence of |s| and ϕ_s (1 Port, Lossy)

Testing a generalization of Dyson's Circular Ensemble Hypothesis

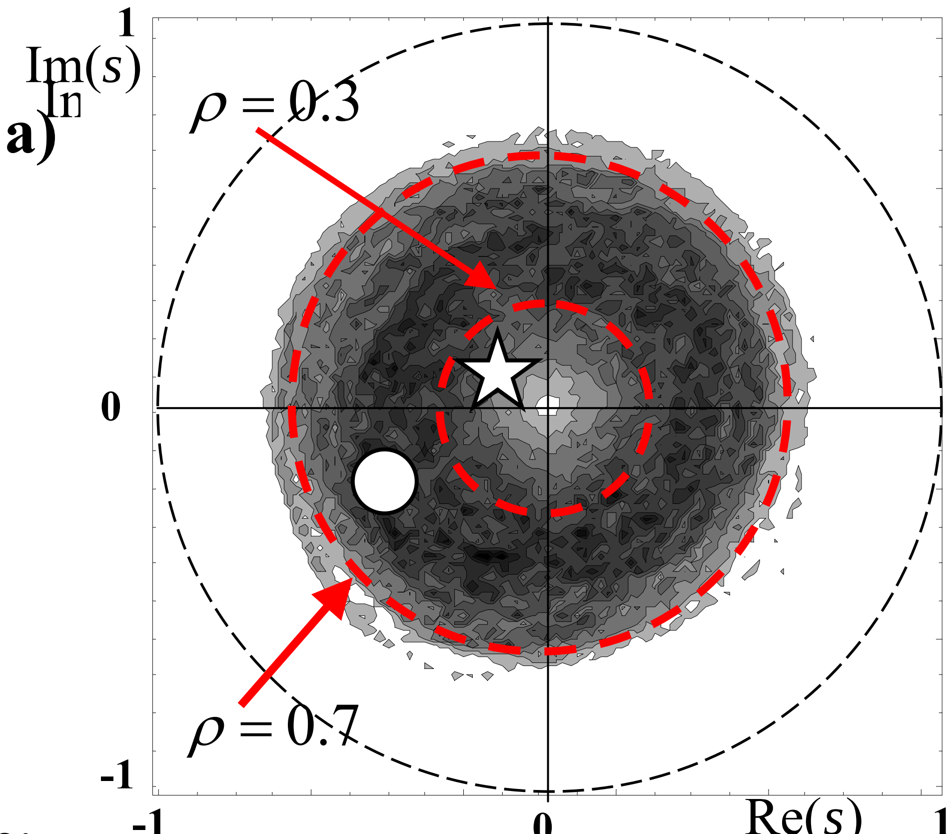
$$s = \frac{1 - S^*_{Rad}}{1 + S_{Rad}} \frac{S - S_{Rad}}{1 - S S^*_{Rad}}$$

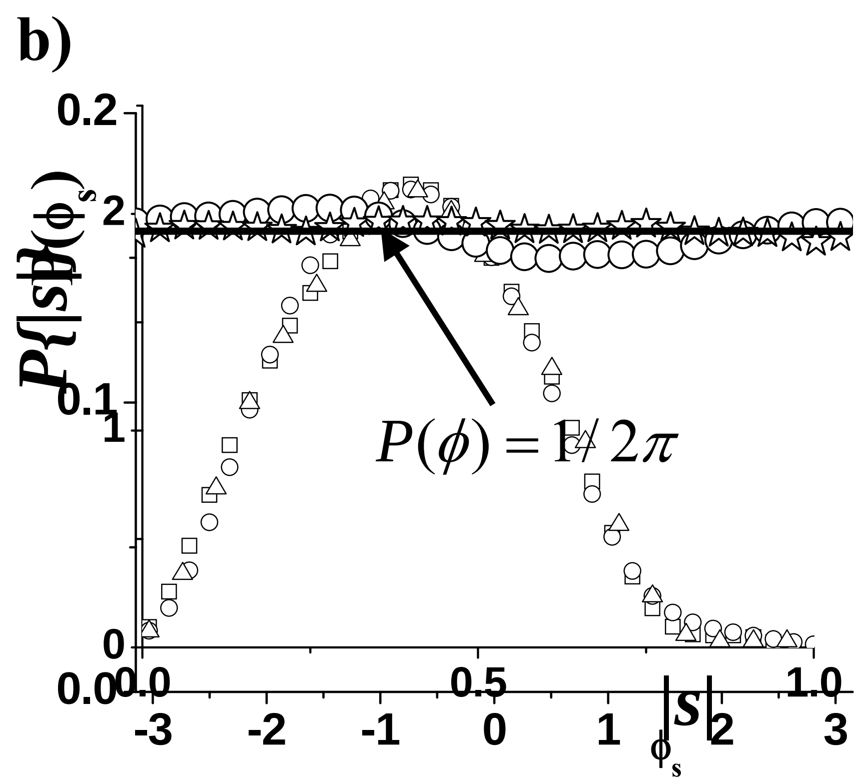
or $$s = \frac{z-1}{z+1}$$

- Frequency: 6 to 9.6 GHz
- Cavity Height : 7.87mm
- Antenna Diameter : 1.27mm



S. Hemmady, *et al.*, Phys. Rev. E71, 056215 (2005)



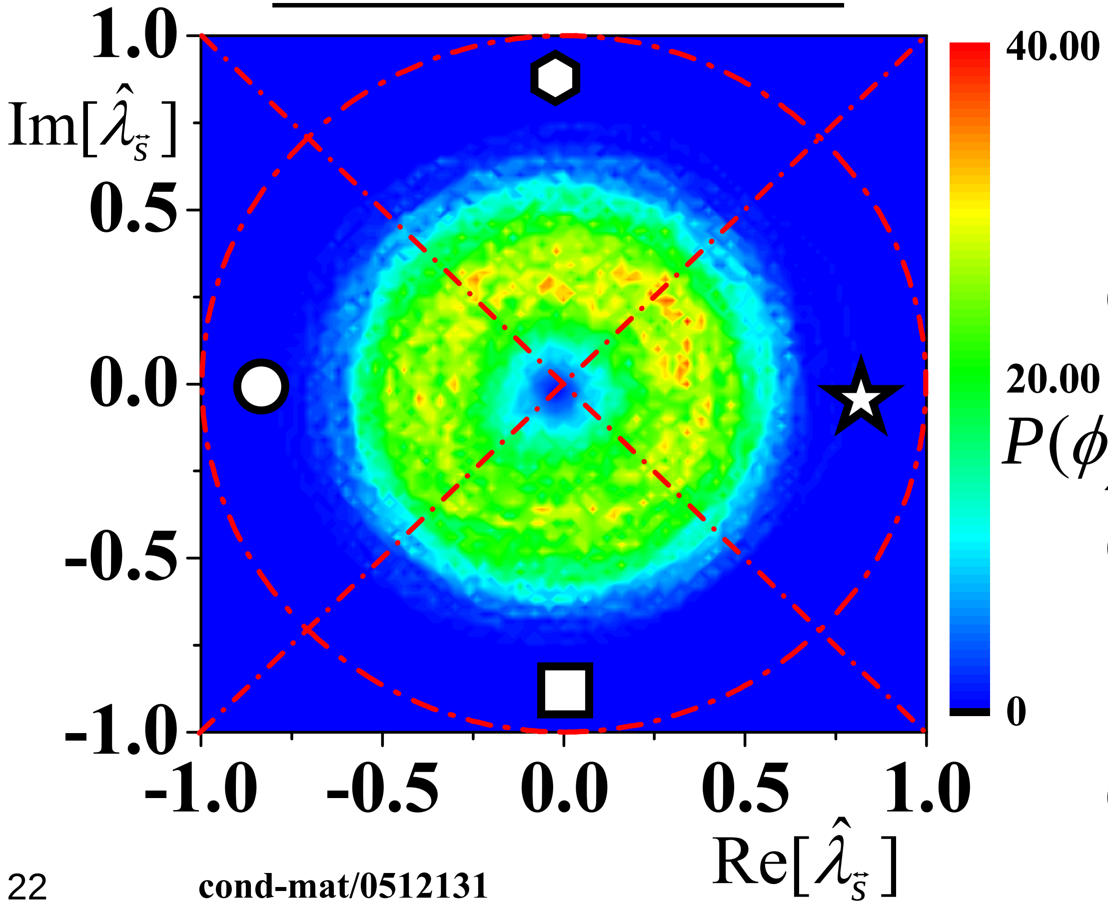


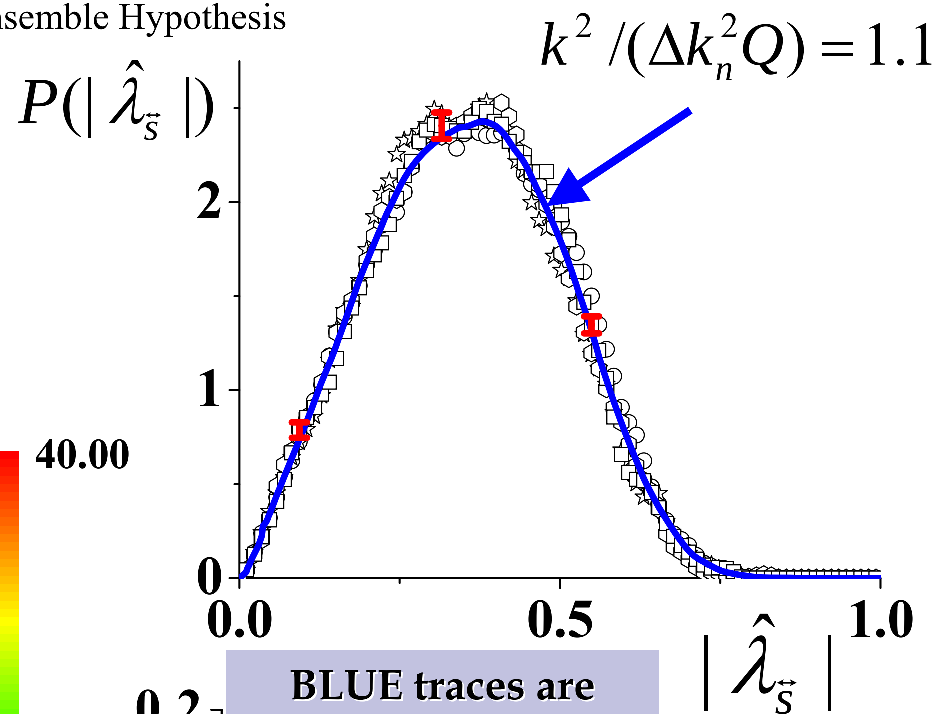
Statistical Independence of |s| and ϕ_s (2 Port, Lossy)

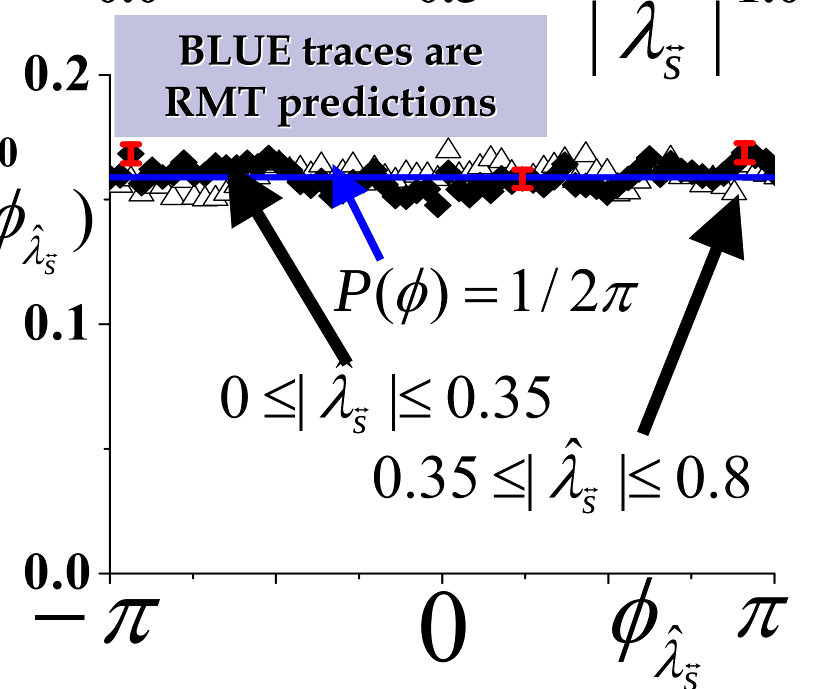
Testing a generalization of Dyson's Circular Ensemble Hypothesis

$$\vec{\vec{s}} = \vec{\vec{V}} \begin{pmatrix} |\lambda_1| Exp[j\phi_1] & 0 \\ 0 & |\lambda_2| Exp[j\phi_2] \end{pmatrix} \vec{\vec{V}}^{-1}$$

Loss Case 0 : 7.6 to 8.1 GHz





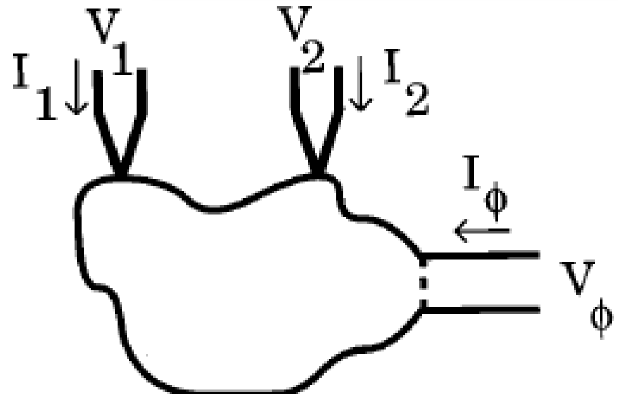


cond-mat/0512131

PHYSICAL REVIEW B VOLUME 55, NUMBER 7 15 FEBRUARY 1997-I

Voltage-probe and imaginary-potential models for dephasing in a chaotic quantum dot

P. W. Brouwer and C. W. J. Beenakker
Instituut-Lorentz, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands



$$S=\begin{pmatrix} s_{11} & s_{12} & s_{1\phi} \\ s_{21} & s_{22} & s_{2\phi} \\ s_{\phi 1} & s_{\phi 2} & s_{\phi\phi} \end{pmatrix} \rightarrow s = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$ sub-unitary

Polar Decomposition of s :

$$s = u\begin{pmatrix} \sqrt{1-T_1} & 0 \\ 0 & \sqrt{1-T_2} \end{pmatrix} u'$$

$1-T_1$ and $1-T_2$ are eigenvalues of $ss^\dagger$

T_1 and T_2 give the "absorption strength" of the fictitious lead ($0 \le T_1, T_2 \le 1$)

$$N_\phi \to \infty, \; \Gamma_\phi \to 0$$
$$\gamma = N_\phi \Gamma_\phi$$

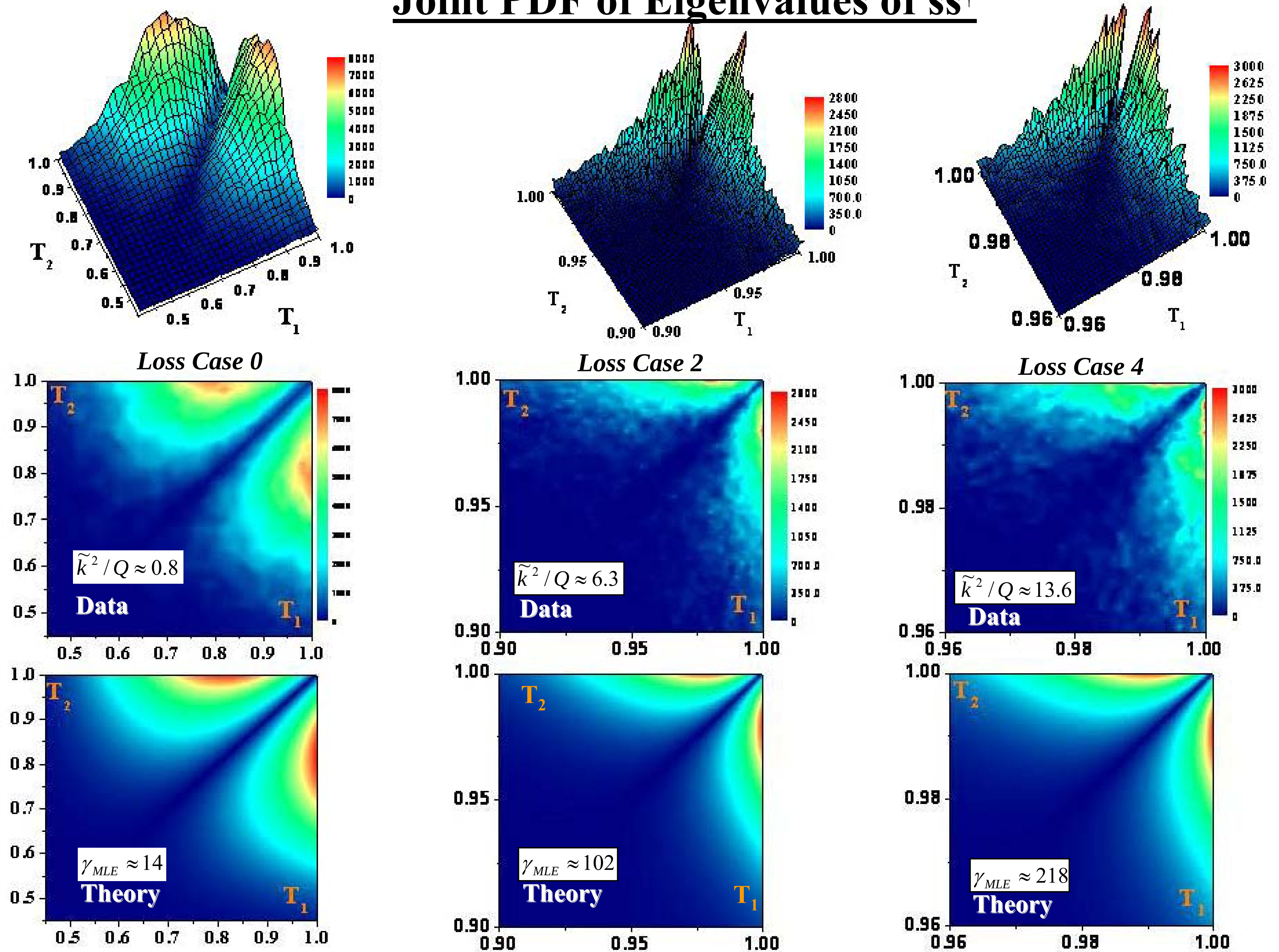
Locally weak absorption limit
Zirnbauer Nuc. Phys. A '93

Buttiker PRB '86
Marcus PRL '92
Baranger + Mello, PRB '95

$$\begin{aligned} P(T_1,T_2) = {} & \tfrac{1}{8} T_1^{-4} T_2^{-4} \exp[-\tfrac{1}{2}\gamma(T_1^{-1}+T_2^{-1})]\,|T_1 - T_2|\,[\gamma^2(2-2e^\gamma+\gamma+\gamma e^\gamma) \\ & -\gamma(T_1+T_2)(6-6e^\gamma+4\gamma+2\gamma e^\gamma+\gamma^2) \\ & +T_1T_2(24-24e^\gamma+18\gamma+6\gamma e^\gamma+6\gamma^2+\gamma^3)] \end{aligned}$$

Joint distribution
for T_1 and T_2 for $\beta = 1$
One mode in each lead
$0 \le T_1, T_2 \le 1$

Joint PDF of Eigenvalues of $ss^\dagger$



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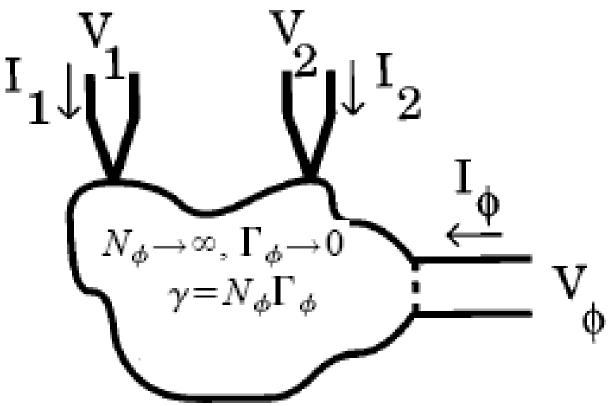
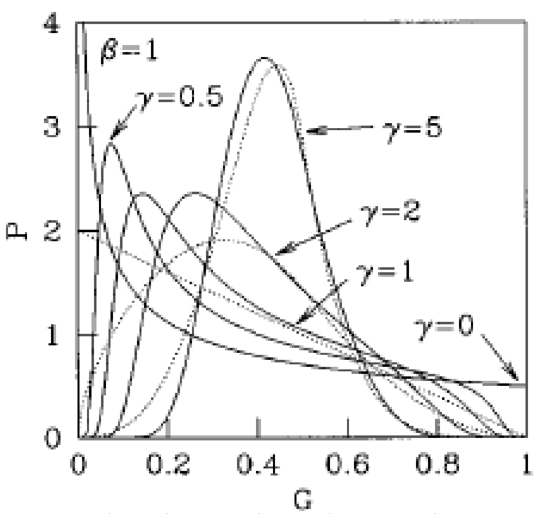


$$G/G_0 = |S_{12}|^2 + \frac{\left(1-|S_{11}|^2-|S_{12}|^2\right)\left(1-|S_{22}|^2-|S_{12}|^2\right)}{\left(2-|S_{11}|^2-|S_{22}|^2-|S_{12}|^2-|S_{21}|^2\right)}$$

$|S_{12}|^2$: Schanze, *et al.* PRB 2005



Numerical evaluation of P(G)

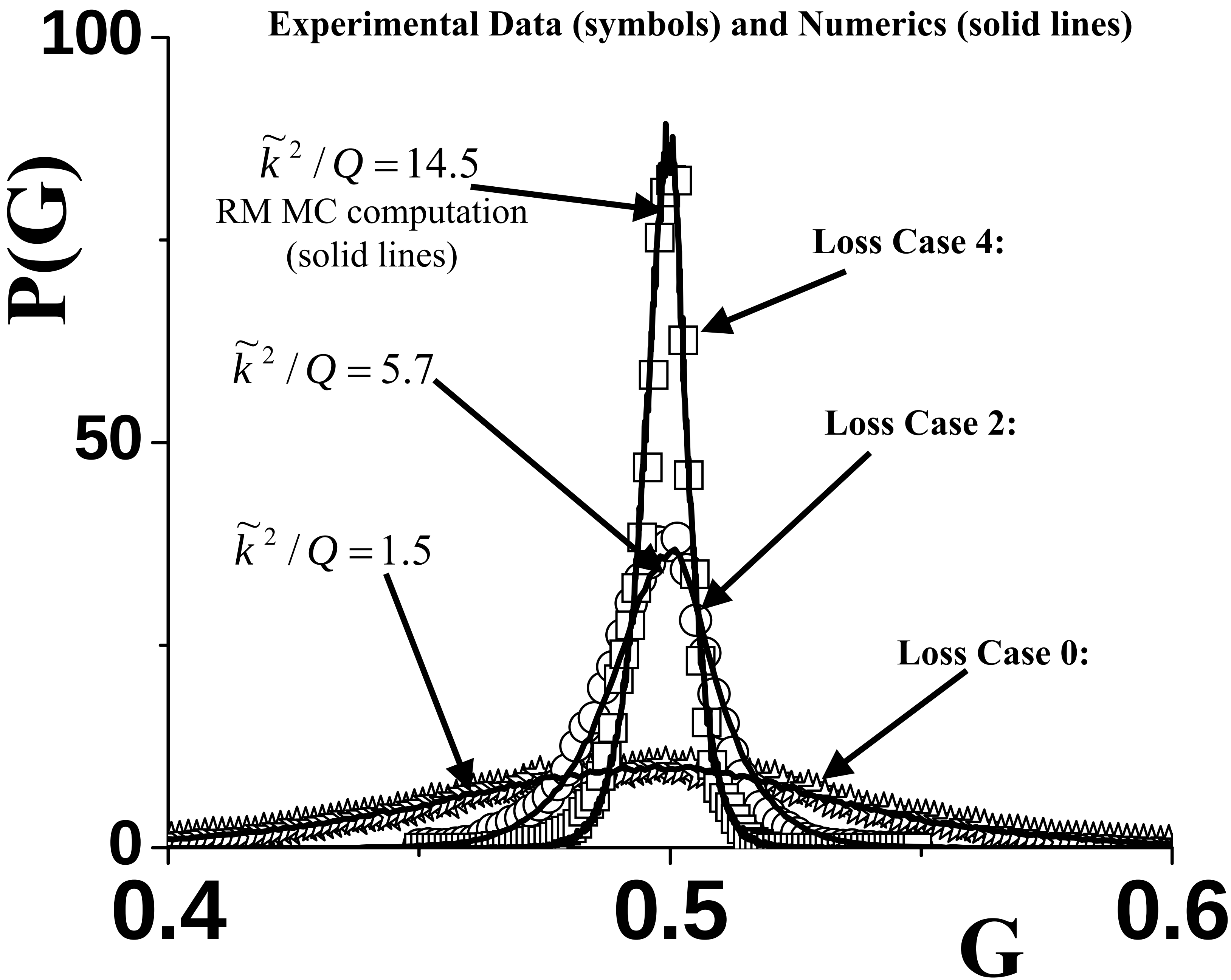
$$P(G) \to \frac{\gamma\beta}{2}(1+|x|-\delta_{\beta 1}x)e^{-|x|} \quad \text{if} \quad \gamma \gg 1$$

$$x = 2\gamma\beta(G - \frac{1}{2})$$

$$\langle G \rangle = \tfrac{1}{2} - \tfrac{1}{2}\delta_{\beta 1}\gamma^{-1} + O(\gamma^{-2}),$$

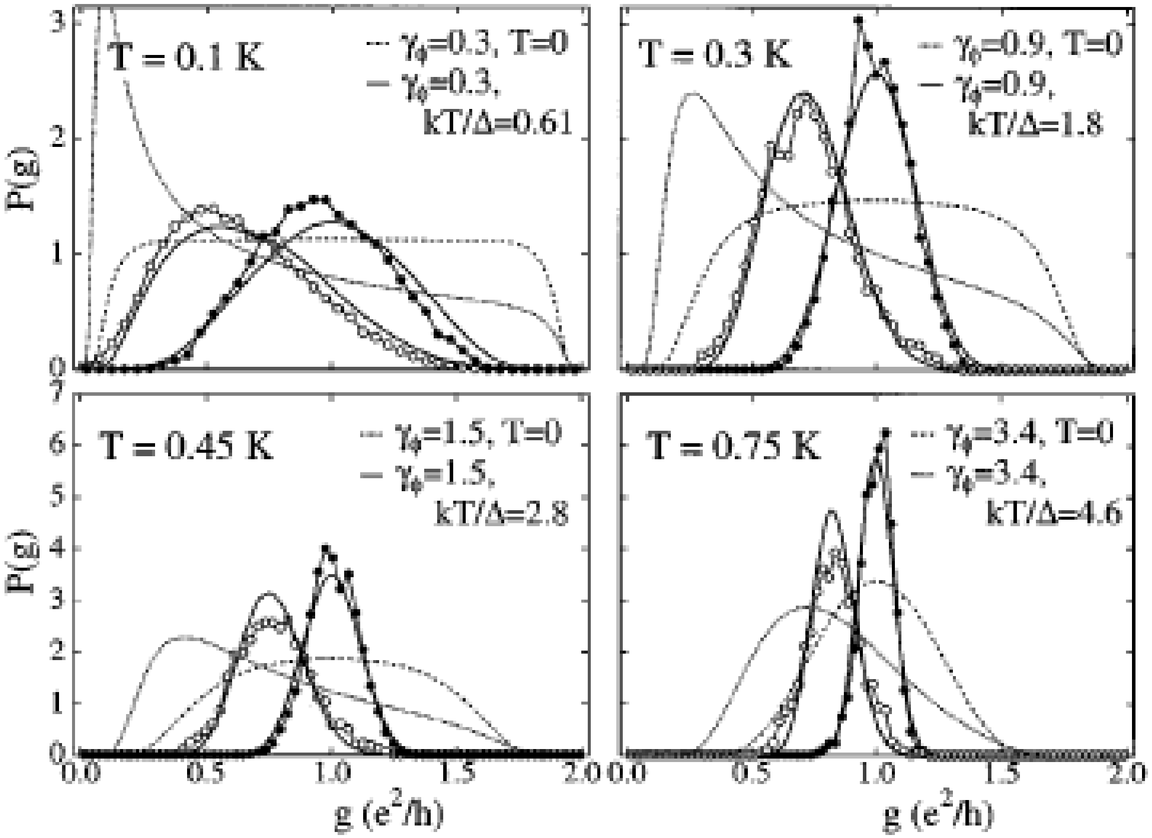
$$\mathrm{var}\,G = \tfrac{1}{4}(1+2\delta_{\beta 1})\gamma^{-2} + O(\gamma^{-3})$$

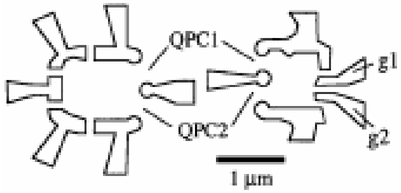
Conductance Fluctuations

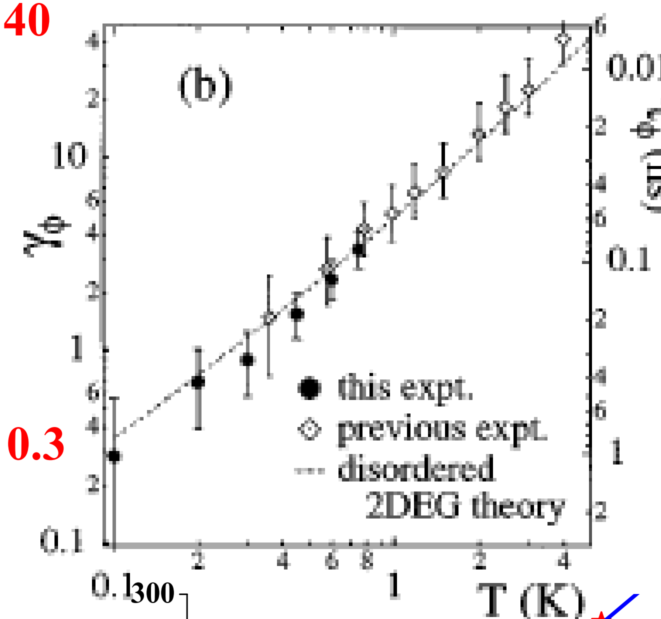


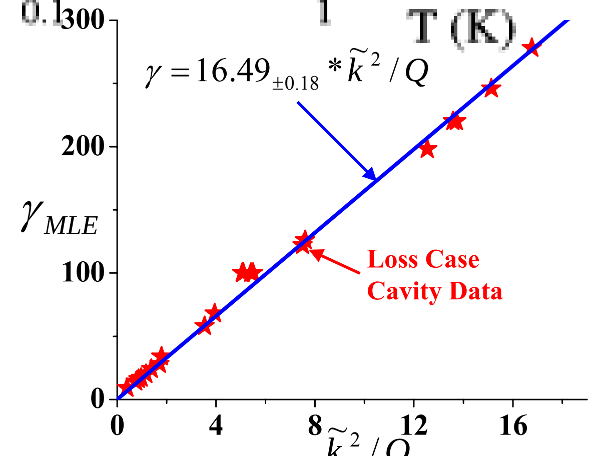
Experimental Conductance Distributions in Real Quantum Dots

A. G. Huibers, *et al.*, PRL 81, 1917 (1998)







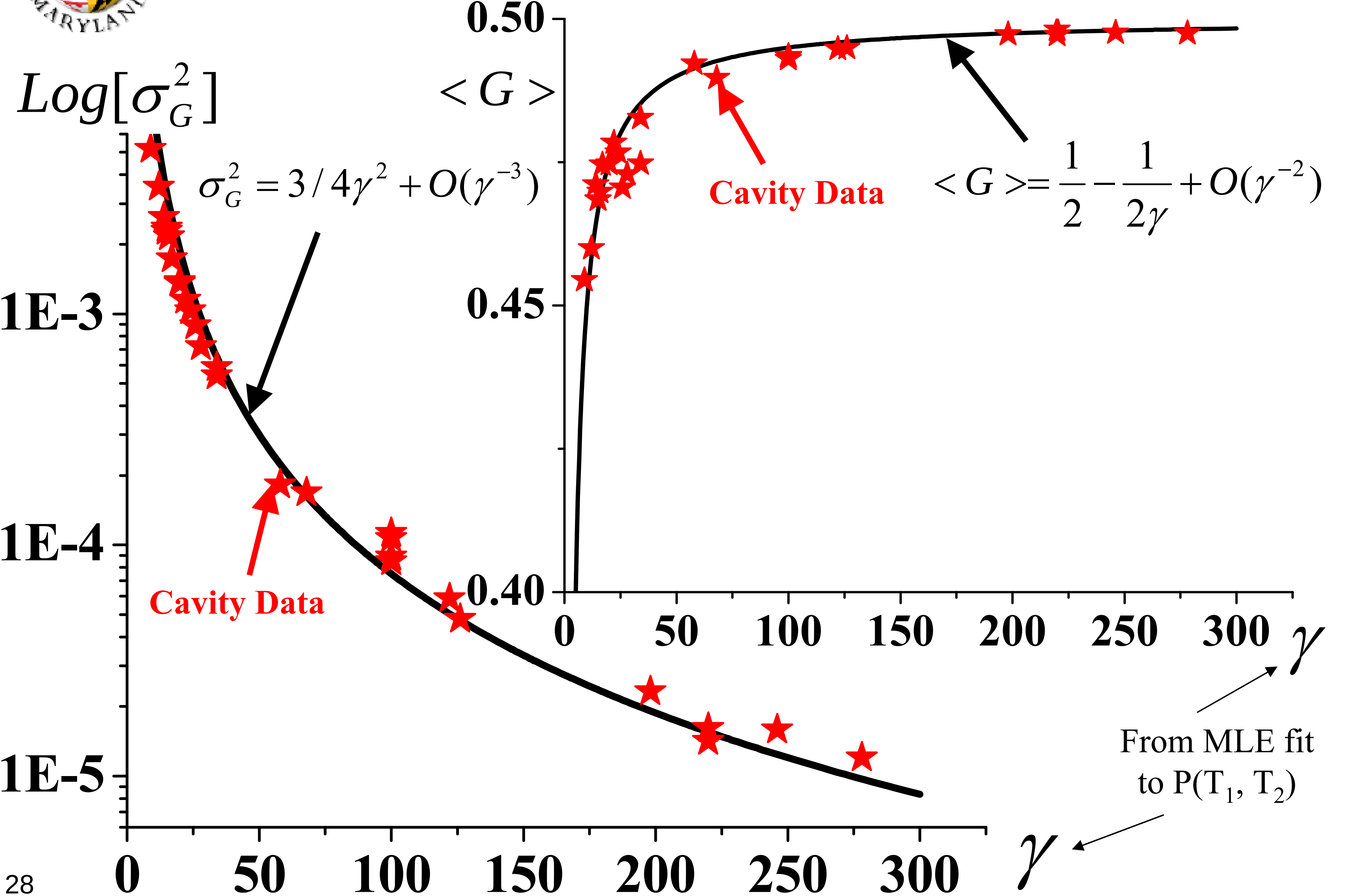


Combined effects of de-phasing
+ thermal broadening

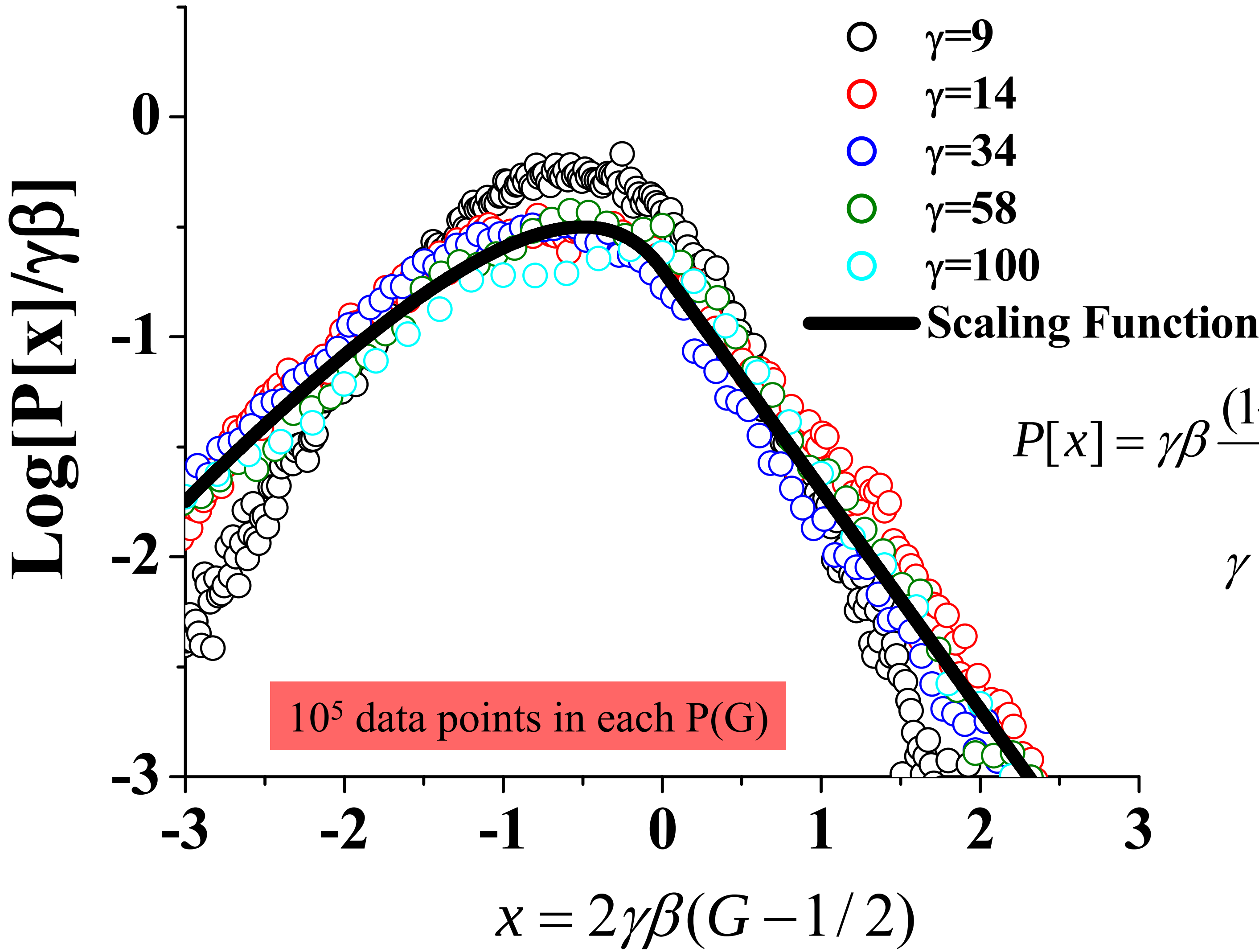
Our data has γ ~ 9 to 300

$5\pi^2/3$

Comparison of Cavity P(G) Statistics with B+B's Asymptotic Formulae



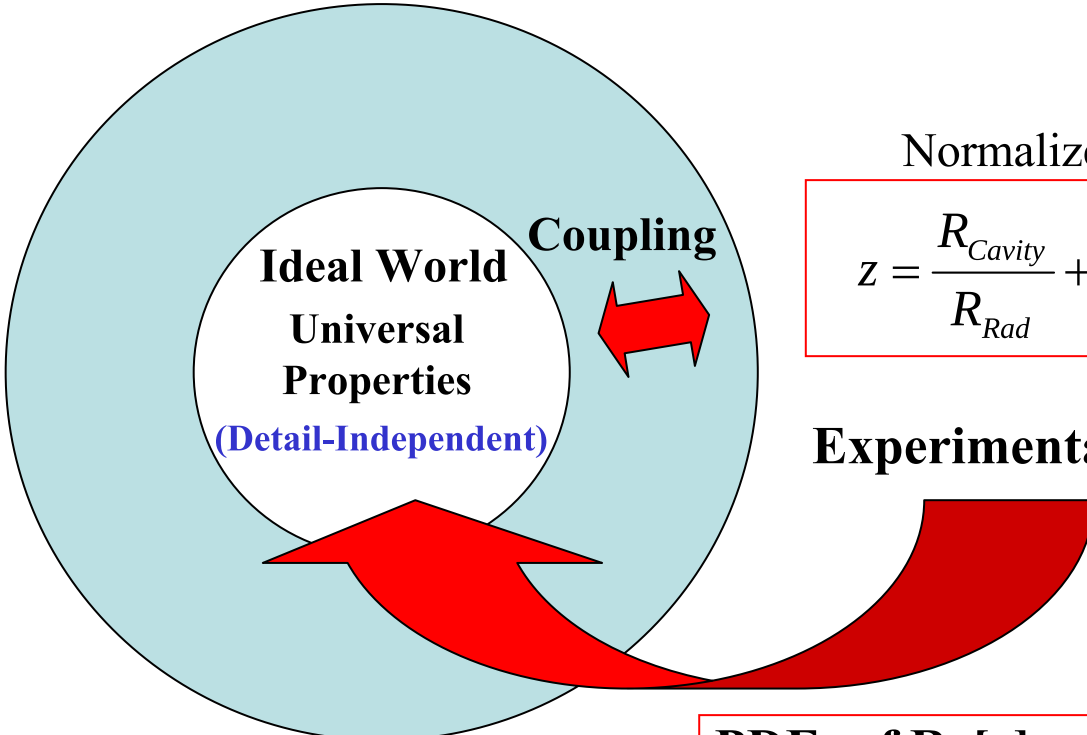
High-Loss Scaling Function for Universal Conductance Fluctuations



$$P[x] = \gamma\beta\,\frac{(1 + |x| - x)e^{-|x|}}{2}$$

$$\gamma >> 1$$

Review of Testable Predictions (1-Port)



Normalized impedance

$$z = \frac{R_{Cavity}}{R_{Rad}} + j\,\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

Experimental Data

Non-Universal

(Detail-dependent)

PDFs of Re[z] and Im[z] are universal

$$\sigma^2\{\mathrm{Re}[z]\} \cong \sigma^2\{\mathrm{Im}[z]\} \cong (1/\pi)\Big/\left(\frac{k^2}{\Delta k_n^2 Q}\right)$$

For TRS Systems

(Q >> 1, β = 1)

Universal variance ratio for Z

$\overline{|S_{11}|^2}$ depends only on $\tilde{k}^2/Q$ and $|S_{11,\mathrm{Rad}}|$

Normalized Scattering Parameter

$$s = \frac{z-1}{z+1}$$

|s| and Arg[s] are statistically independent

PRL <u>94</u>, 014102 (2005); PRE <u>71</u>, 056215 (2005); cond-mat/0504196; cond-mat/0512131

Conclusions

Deterministic measurements of the radiation impedance remove the effects of direct processes to recover universal statistical electromagnetic properties

Experimental tests of many basic 1 port and 2-port predictions have confirmed that the approach is correct.

Universal statistical properties of s-matrices

- Independence of magnitude and phase of eigenvalues
- Joint PDFs of eigenvlaue phases, magnitudes
- Conductance fluctuations in good agreement with theory

S. Hemmady, *et al*., Phys. Rev. Lett. 94, 014102 (2005)
X. Zheng, *et al*., J. Electromag., in press: cond-mat/0408317
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S. Hemmady, *et al*., cond-mat/0512131

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