Superuniversal Statistics of Complex Time Delays in Non-Hermitian Scattering Systems

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The Wigner-Smith time delay of flux conserving systems is a real quantity that measures how long an excitation resides in an interaction region. The complex generalization of time delay to non-Hermitian systems is still under development, in particular, its statistical properties in the short-wavelength limit of complex chaotic scattering systems has not been investigated. From the experimentally measured multiport scattering (S) matrices of one-dimensional graphs, a two-dimensional billiard, and a three-dimensional cavity, we calculate the complex Wigner-Smith (τ_{WS}), as well as each individual reflection (τ_{xx}) and transmission (τ_{xy}) time delay. The complex reflection time-delay differences ($\tau_{\delta R}$) between each port are calculated, and the transmission time-delay differences ($\tau_{\delta T}$) are introduced for systems exhibiting nonreciprocal scattering. Large time delays are associated with scattering singularities such as coherent perfect absorption, reflectionless scattering, slow light, and unidirectional invisibility. We demonstrate that the large-delay tails of the distributions of the real and imaginary parts of each time-delay quantity are superuniversal, independent of experimental parameters: wave propagation dimension $\mathcal{D}$, number of scattering channels M, Dyson symmetry class β, and uniform attenuation η. The tails determine the abundance of the singularities in generic scattering systems, and the superuniversality is in direct contrast with the well-established time-delay statistics of unitary scattering systems, where the tail of the τ_{WS} distribution depends explicitly on the values of M and β. We relate the distribution statistics to the topological properties of the corresponding singularities. Although the results presented here are based on classical microwave experiments, they are applicable to any non-Hermitian wave-chaotic scattering system in the short-wavelength limit, such as optical or acoustic resonators.

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Introduction—In this Letter, we consider the wave-scattering properties of generic complex systems with a finite number of asymptotic scattering channels coupled to the outside world. The systems of interest have dimensions much larger than the wavelength of the waves, making the scattering properties extremely sensitive to details, such as boundary shapes and interior scattering centers. Such systems could be three-dimensional spaces such as rooms, two-dimensional billiards, or one-dimensional graphs, and the waves could be electromagnetic, acoustic, or quantum in nature. The complexity of wave propagation and interference is captured by the scattering S matrix which transforms a vector of input excitations $|\psi_{\mathrm{in}}\rangle$ on M channels to a vector of outputs $|\psi_{\mathrm{out}}\rangle$ as $|\psi_{\mathrm{out}}\rangle = S|\psi_{\mathrm{in}}\rangle$. The scattering diversity gives rise to strong dependence of the complex S-matrix elements as a function of excitation energy E. We focus in particular on the time these waves spend in the scattering region as they propagate from one scattering channel to another. An average dwell time was introduced by Wigner [1] in the context of nuclear scattering. Smith [2] later generalized this idea by inventing a lifetime matrix $Q = i\hbar S(dS^{\dagger}/dE)$, the normalized trace of which is defined as the Wigner-Smith time delay $\tilde{\tau}_{\mathrm{WS}} = (1/M)\mathrm{Tr}[Q]$, which is strictly real for unitary (flux-conserving) systems.

For a quantum mechanical wave, $\tilde{\tau}_{\mathrm{WS}}$ can be directly related to the phase evolution of a wave packet and its group delay [3,4]. For classical waves, the Wigner-Smith time delay is simply the energy-dependent shift in arrival time caused by the scattering interaction [5]. Some examples of the physical uses of time delay are in nuclear physics [6,7], photoionization [8–12], tunneling time [13,14], group delay of modes in waveguides and optical fibers [15–19], wave front shaping and creation of particlelike scattering states [20–24], radio frequency pulse propagation [25,26], radiation intensity statistics [27,28], identifying zeros and poles of the scattering matrix [29–34], characterization of disordered and biological media [35–40], and acoustics [41].

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When studying scattering of short-wavelength excitations in complex systems, such as nuclei or irregular electromagnetic structures, it is appropriate to examine the statistical properties of time delays, due to the sensitive energy and parametric variations of the scattering process. Random matrix theory (RMT) has been very successful at describing universal fluctuations of time delays in unitary scattering systems [29,42–55]. This body of work discovered that the probability distribution $\mathcal{P}(\tilde{\tau}_{\mathrm{WS}})$ has certain consistent properties. One of particular interest is the shape of $\mathcal{P}(\tilde{\tau}_{\mathrm{WS}})$ at extreme values, which correspond to modes with long scattering times. It was found independently by several groups that the distribution of the Wigner-Smith time delay of a perfectly coupled system has an algebraic power-law decay for large values of the time delay described by $\mathcal{P}(\tilde{\tau}_{\mathrm{WS}}) \propto \tilde{\tau}_{\mathrm{WS}}^{-(M\beta)/2-2}$ [56–63], where β is the Dyson symmetry class of the system [64]. A more complete review of Wigner-Smith time-delay results and analysis in unitary scattering systems can be found in Ref. [65].

Physical systems have loss and/or gain, and non-Hermitian effects cannot be ignored in experimental scattering data [38,66–75]. For weak attenuation, an expansion about the unitary scattering matrix is possible [76–79], but the most general way to account for dissipation leads to a complex $\tilde{\tau}_{\mathrm{WS}}$ [5,24,33,80]. Even when the scattering matrix is unitary, the reflection and transmission submatrices are subunitary, leading to complex time delays (CTDs) describing reflection and transmission processes [5,81]. The reflection time-delay difference between channels has been proposed as a quantity that depends only on the reflection zeros of the system [31,32,34]. We introduce here the complex *transmission time-delay difference* in nonreciprocal scattering systems, which depends solely on the system's *transmission* zeros.

The presence of absorption introduces qualitatively new phenomena not seen in unitary systems, such as coherent perfect absorption (CPA), in which all the energy injected into a system is completely absorbed, with no reflection or transmission on any of the scattering channels [23,24,30,82–86]. Also known as antilasing, this phenomenon has drawn interest in optics [87–89], acoustics [90,91], heat transfer [92], and quantum single photon systems [93–95]. CPA has been associated with diverging Wigner-Smith time delay [24,96], justifying attention to the large values of time delay. While $\tilde{\tau}_{\mathrm{WS}}$ features most prominently in the literature, the other time delays should not be forgotten, as, for example, divergences of the reflection time delay correspond to reflectionless scattering modes (RSM) [86,97,98], and the divergences of the transmission time-delay difference could identify points of unidirectional invisibility [99–101]. In Fig. 1, we show the real parts of the Wigner Smith, reflection, and transmission time delay from an experimental microwave graph (discussed below) over a range of frequency and phase shift along one bond. The purple symbols mark the divergences of the time delays and their corresponding singularities. When looking at a probability distribution of delay from a statistical ensemble, the tail of the distribution gives a measure of the abundance of these extreme phenomena, which show rich potential for applications.

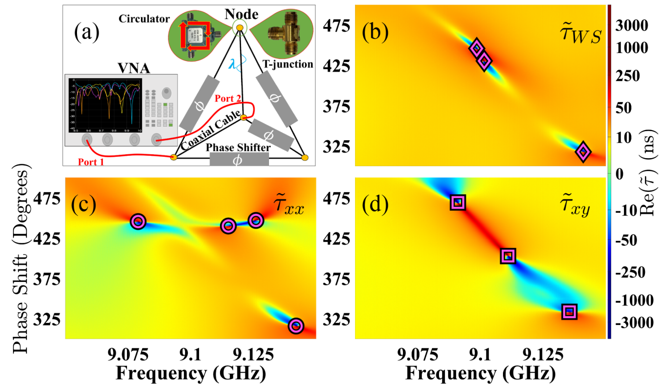

FIG. 1. (a) Schematic of an experimental $M = 2$ tetrahedral graph ($\mathcal{D} = 1$). Four of the six bonds of the graph are comprised of phase shifters. Each node can be an SMA T-junction or microwave circulator. Two-parameter heatmap of (b) $\mathrm{Re}[\tilde{\tau}_{\mathrm{WS}}]$, (c) $\mathrm{Re}[\tilde{\tau}_{xx}]$, and (d) $\mathrm{Re}[\tilde{\tau}_{xy}]$ from a reciprocal ($\beta = 1$) tetrahedral microwave graph with $\Delta = 46$ MHz and $\tau_H = 21$ ns. Purple symbols indicate locations of diverging CTD at (b) CPAs, (c) zero reflection, and (d) zero transmission points.

Recently, Chen *et al.* showed that $\mathcal{P}(\mathrm{Re}[\tilde{\tau}_{\mathrm{WS}}])$ and $\mathcal{P}(\mathrm{Im}[\tilde{\tau}_{\mathrm{WS}}])$ of perfectly coupled $M = 2$ channel, nonreciprocal ($\beta = 2$) graphs ($\mathcal{D} = 1$) with finite uniform absorption strength η have algebriac -3 power-law tails. They predicted that this should be true of $\mathcal{P}(\tilde{\tau}_{\mathrm{WS}})$ for any perfectly coupled non-Hermitian system [102]. In this Letter, we examine the statistics of each CTD with a focus on the asymptotic behavior of their distributions. We consider four global parameters which could dictate the distribution tails: (i) dimension for wave propagation $\mathcal{D}$, (ii) number of channels M, (iii) Dyson class β, and (iv) uniform dissipation η.

We first provide the theory and definitions of CTD in non-Hermitian systems, then present the systems experimentally investigated, and finally we discuss the superuniversal -3 power-law tail of the CTD probability distributions.

Theory—In the Heidelberg approach to wave scattering [29,103–107], the non-Hermitian effective Hamiltonian of a scattering environment $\mathcal{H}_{\mathrm{eff}} = H - i\Gamma_W$ is constructed from the $N \times N$ Hamiltonian H describing the closed system and the $N \times M$ matrix W of coupling coefficients between the N modes of H and the M scattering channels, where $\Gamma_W = \pi W W^{\dagger}$. The energy-dependent scattering matrix $S(E)$ takes the form [29,104,106]

$$S(E) = 1_{M\times M} - 2\pi i W^{\dagger} \frac{1}{E - \mathcal{H}_{\mathrm{eff}}} W = \begin{pmatrix} S_{11} & \dots & S_{1M} \\ \vdots & \ddots & \vdots \\ S_{M1} & \dots & S_{MM} \end{pmatrix}.$$

One can account for a spatially uniform attenuation of the waves with rate $\tilde{\eta}$ by making the substitution $E \to E + i\tilde{\eta}$ and evaluating the resulting subunitary S matrix at complex energies. Experimentally, such a subunitary scattering matrix for an M-channel system is measured as $S(E)$ in terms of M^2 energy-dependent complex reflection (S_{xx}) and transmission (S_{xy}) submatrices where $x \neq y$ ($x, y \in [1, \ldots, M]$) [96,108–110]. It should be noted here that $\tilde{\eta}$ is the uniformly imposed mode bandwidth, and describing uniform loss in this manner is phenomenological, making no special assumptions about the S matrix [106,111,112].

The Wigner-Smith time delay extended to non-Hermitian systems has a natural definition as a complex quantity [33]:

$$\tilde{\tau}_{\mathrm{WS}} \coloneqq \frac{-i}{M} \frac{\partial}{\partial E} \log[\det S(E + i\tilde{\eta})]. \tag{1}$$

$\tilde{\tau}_{\mathrm{WS}}$ can also be written as a sum of Lorentzian functions of energy in terms of the S-matrix poles and zeros, which are the complex eigenvalues of $\mathcal{H}_{\mathrm{eff}}$ and $\mathcal{H}_{\mathrm{eff}}^{\dagger}$, respectively [34,113].

While $\tilde{\tau}_{\mathrm{WS}}$ takes into account the entire scattering matrix, it is also important to consider the time delays of individual reflection and transmission processes [5,80,81,114–118]. The CTDs for reflection at channel x and transmission to channel x from channel y are given by [34]

$$\tilde{\tau}_{xx} \coloneqq -i \frac{\partial}{\partial E} \log[S_{xx}] \tag{2a}$$

$$\tilde{\tau}_{xy} \coloneqq -i \frac{\partial}{\partial E} \log[S_{xy}] \tag{2b}$$

in direct analogy with $\tilde{\tau}_{\mathrm{WS}}$, and can also be written as sums of Lorentzian functions of energy [34,119].

Since the Lorentzian terms arising from the poles are identical for each reflection and transmission time delay, we can define new quantities, the reflection and transmission time-delay *differences*, which are expected to be independent of the S-matrix poles [31,32,34]:

$$\tilde{\tau}_{\delta R} \coloneqq \tilde{\tau}_{xx} - \tilde{\tau}_{yy} = -i \frac{\partial}{\partial E} \log\left[\frac{S_{xx}}{S_{yy}}\right] \tag{3a}$$

$$\tilde{\tau}_{\delta T} \coloneqq \tilde{\tau}_{xy} - \tilde{\tau}_{yx} = -i \frac{\partial}{\partial E} \log\left[\frac{S_{xy}}{S_{yx}}\right]. \tag{3b}$$

In an arbitrary system, usually $S_{xx} \neq S_{yy}$ so $\tilde{\tau}_{\delta R}$ is almost always nonzero. In contrast, to have a nontrivial $\tilde{\tau}_{\delta T}$, the system must have nonreciprocal transmission. This can be accomplished, for example, with electromagnetic waves propagating through magnetized ferrite materials [120–129]. The transmission time-delay difference written as sums of Lorenztian functions of energy is given for the first time in Eqs. (S2) and (S3) in Supplemental Material (SM) [130].

In highly overmoded structures, CTDs fluctuate strongly in energy (or equivalently frequency f), with both the real and imaginary parts taking on positive and negative values [5,24,33,34] [see Fig. S4(a) for a representative example]. Since CTD is very sensitive to perturbations of an overmoded scattering system, we examine the distribution of time delays from a statistical ensemble of many similar systems to make general statements.

Experiment—Microwave experiments have proven to be ideal platforms for investigating fluctuations in wave scattering properties of complex systems [106,132–138]. The experimental data presented in this Letter comes from $M \times M$ scattering matrices S collected through the use of calibrated Keysight PNA-X N5242A and PNA-X N5242B microwave vector network analyzers. The measured scattering parameters from the PNA contain information about the coupling of the system, as well as direct processes (which include short orbits between ports). These nonuniversal effects are removed from an ensemble through application of the random coupling model (RCM) normalization process, which results in an S matrix with perfect coupling, and reduces the effects of short orbits on the statistics [69–71,75,137,139–147]. To make a statistical ensemble, tunable perturbers that can change local conditions between realizations are embedded in the scattering systems. High quality determinations of η are extracted from the statistical ensembles; details can be found in SM [130] Sec. S1C.

Because physical systems have system-specific shape and size, the value in seconds of any time-delay $\tilde{\tau}$ is dependent to some extent on irrelevant details. To isolate the universally fluctuating properties, it is necessary to normalize $\tilde{\tau}$ by the Heisenberg time $\tau_H = (2\pi/\Delta)$, where Δ is the mean mode spacing of the closed system [102,148,149]. The scaled time delays $\tau \coloneqq \tilde{\tau}/\tau_H$ discussed in this Letter are dimensionless quantities independent of the specific system measured. We also normalize the absorption rate of the system to a dimensionless quantity in units of the Heisenberg time $\eta \coloneqq \tau_H \tilde{\eta}$.

Our $\mathcal{D} = 1$ system is an irregular tetrahedral graph, a well-studied graph topology [83,150–156], with tunable phase shifters on the bonds to manipulate the wave interference conditions [146,157–160], depicted schematically in Fig. 1(a). For $\mathcal{D} = 2$ and $\mathcal{D} = 3$ we used a two-dimensional ray-chaotic 1/4-bow-tie billiard [121,133,138,161,162] and a three-dimensional ray-chaotic cavity [23,163–169], each containing voltage controlled, varactor-loaded metasurfaces which can significantly alter the boundary conditions of the systems [85,169–172]. A schematic of the billiard used in the experiment is shown in Fig. 1 of Ref. [85], and a photograph of the cavity and metasurface is given in Fig. S2 of the SM [130]. The systems were measured with $M = 1$, 2, 3 scattering channels. The Dyson symmetry class β of a system can be changed from 1 to 2 through inclusion of a magnetized ferrite in the microwave propagation path, such as including a circulator at one of the nodes in a graph [125–127,173–176]. We also consider a graph that has a circulator on every node connected to a scattering channel, but nowhere else (see Fig. S1 in SM [130]), which we term “nonreciprocal coupling” (NRC), as a special case of $\beta = 2$. Each system has its own intrinsic uniform absorption strength η which is frequency dependent [34]; therefore, by measuring in different frequency bands we can systematically vary the value of η. The uniform attenuation can also be increased by uniformly distributing absorbers in a cavity [72,74,139,177] or attenuators on the bonds of a graph [128,160,178]. The absorption rate of the systems considered here varied from $\eta = 1.8$ to $\eta = 50$, creating data in what are considered the low, moderate, and high absorption strength regimes [66,67,128,139,151,179,180].

More details on the specific experimental systems as well as the creation of statistical ensembles can be found in SM [130] Sec. S1.

Discussion—We present the probability distribution functions (PDFs) of the CTDs calculated using Eqs. (1)–(3b) from the measured ensembles of S-matrix data that have been RCM normalized to establish perfect coupling at all frequencies.

Figure 2(a) shows the distributions of $\text{Re}[\tau_{\text{WS}}]$ for five systems with different values for the four experimental parameters, $\mathcal{D}$, M, β, η, and Fig. 2(b) shows the PDFs of $|\text{Im}[\tau_{\text{WS}}]|$. The black lines are not fits, but -3 power laws placed nearby to characterize the large-delay tail behavior. For large values of τ_{WS}, all distributions have parallel tails with the same slope as the black reference line. The dashed red line in Fig. 2 depicts a -4 power-law, which a Hermitian $\beta = 2$, $M = 2$ system would have, but clearly deviates from the data.

Individual complex reflection and transmission time-delay distributions calculated from Eqs. (2a) and (2b) are presented in Supplemental Material Sec. S3 [130]. The tails of these distributions, both real and imaginary, have a -3 power-law. We also look at the distributions of the CTD differences, as defined in Eqs. (3a) and (3b). Figure S7 in SM [130] shows $\mathcal{P}(\tau_{\delta R})$ of five systems with varying parameters. Figure 3 shows the distributions of the new quantity $\tau_{\delta T}$ for six systems with $\mathcal{D} = 1$ and $\beta = 2$. For this work all measured broken-reciprocity systems were graphs ($\mathcal{D} = 1$). More detail on $\tau_{\delta T}$ is available in SM [130] Sec. S2.

Across Figs. 2 and 3, and Figs. S5–S7, we show empirical evidence for a superuniversal -3 power-law tail behavior (see Table I for numerical values) for every CTD ($\tau_{\text{WS}}, \tau_{xx}, \tau_{xy}, \tau_{\delta R}, \tau_{\delta T}$) distribution of perfectly coupled non-Hermitian systems. The tail on all these distributions implies that no CTD quantity has a finite variance. Compare this to the established variance of τ_{WS} for Hermitian systems, which diverges only for $M\beta = 2$ [51,62,181].

According to these results, extreme time-delay phenomena and their associated scattering singularities (such as CPAs, RSMs, etc.) are not events that require careful engineering. A complex scattering system with

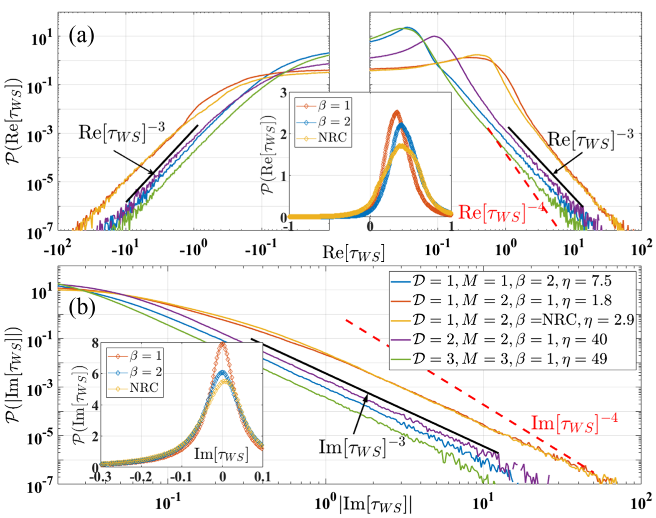

FIG. 2. PDFs of (a) $\text{Re}[\tau_{\text{WS}}]$ and (b) $|\text{Im}[\tau_{\text{WS}}]|$ Wigner-Smith CTD from ensembles of experimental scattering systems with different values for the four parameters $\mathcal{D}$, M, β, η. Black reference line characterizes asymptotic behavior as τ_{WS}^{-3} power law for all distributions. Dashed red line depicts -4 power law showing deviation from distributions. Insets depict representative distributions of a $\beta = 1$, $\beta = 2$, and NRC system on a linear scale, all with $\mathcal{D} = 1$, $M = 2$, and approximately equal η.

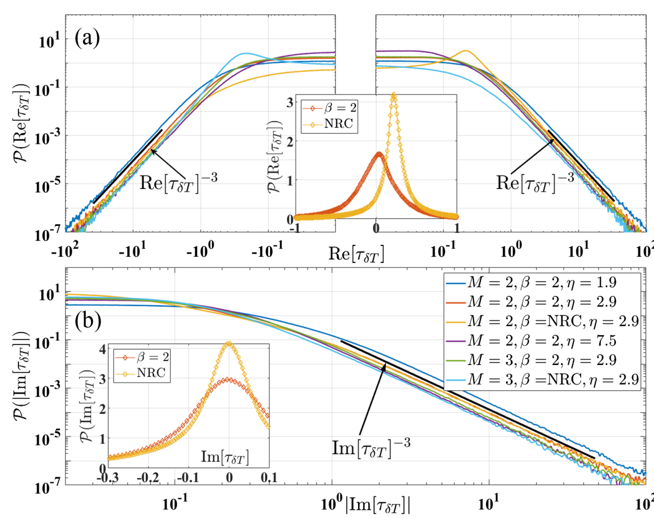

FIG. 3. PDFs of (a) $\text{Re}[\tau_{\delta T}]$ and (b) $|\text{Im}[\tau_{\delta T}]|$ transmission time-delay difference from ensembles of experimental scattering systems with different values for M and η, and two different nonreciprocal ($\beta = 2$, NRC) graphs. All distributions come from graphs ($\mathcal{D} = 1$). Insets and reference lines serve the same purpose as in Fig. 2.

TABLE I. Table of power-law fits to experimental time-delay distributions. First and second columns are the kind of CTD and the number of distributions fit to the form $\mathcal{P}(^{\mathrm{Re}}_{\mathrm{Im}}[\tau]) = {}^{\mathrm{Re}}_{\mathrm{Im}}[\tau]^{-\alpha}$ over the range $10^{-6} \leq \mathcal{P}(^{\mathrm{Re}}_{\mathrm{Im}}[\tau]) \leq 10^{-3}$. The third (fourth) column is the mean fit for tails of the distributions of the real (imaginary) components with uncertainty.

CTD τ	# Distributions fit	α_{Re}	α_{Im}
τ_{WS}	27	2.984 ± 0.006	2.99 ± 0.03
τ_{xx}	49	2.982 ± 0.006	3.00 ± 0.02
τ_{xy}	64	3.010 ± 0.008	3.00 ± 0.014
$\tau_{\delta R}$	32	2.994 ± 0.008	3.030 ± 0.007
$\tau_{\delta T}$	44	3.006 ± 0.005	3.011 ± 0.009

tunable local parameters, such as metamaterials [23,85,86,169–172], should host many such singularities, as was seen to be the case for S-matrix exceptional point degeneracies in [182]. Such controlled perturbations can be accomplished not only with microwave photonics, but also in acoustics [183–185] and optics [82,88,186].

While all the results discussed in this Letter are for electromagnetic waves, the results apply generally to all classical wave scattering systems. It has also been noted that there is a direct analogy between quantum weak measurements and CTD [5,187–189], suggesting that the statistical results presented here extend also to the realm of quantum measurements.

In Supplemental Material [130] Sec. S4 we demonstrate that the -3 power-law tail is also displayed in the distributions of CTD calculated from RMT numerical data, using the same set of parameters used in the experiment. The statistics of time delays derived from S-matrix data with (imperfect) frequency dependent coupling are shown in Supplemental Material [130] Sec. S5B.

Concerning theory, the -3 power-law tails were predicted for $\mathcal{P}(\mathrm{Re}[\tau_{\mathrm{WS}}])$, $\mathcal{P}(\mathrm{Im}[\tau_{\mathrm{WS}}])$ [102], and $\mathcal{P}(\mathrm{Re}[\tau_{\delta R}])$ [31,32] through considerations of the S-matrix poles and zeros for the first two, and reflection zeros for the latter. We detail nonrigorous plausibility extensions of these predictions to $\mathcal{P}(\tau_{xx})$ and $\mathcal{P}(\tau_{\delta T})$ in SM [130] Sec. S6. However, there is no way known to us to do something similar for $\mathcal{P}(\tau_{xy})$ [96,190]. Since there is a–3 power-law tail observed for all time-delay quantities, it can be inferred that there should be an underlying reason, independent of the various poles and zeros of the scattering matrix.

We suggest a possible explanation for the observed, unpredicted superuniversality. The tails of the time-delay distributions essentially describe the relative likelihood of time-delay divergences, which occur at scattering singularities. These singularities are zeros of complex scalar functions, which are topologically stable [191], meaning they do not appear or disappear spontaneously, but only through pairwise creation or annihilation with a partner of opposite winding number [182,192]. In a 2-parameter space [see Figs. 1(b)–1(d)], the time-delay divergences occur at singular points. We propose that the -3 power-law tail is generic to the distributions of any such quantity with these topological defects, including CTD. Another example is the quantum weak measurement value calculated by Solli *et al.* [188], which diverges at topologically stable singularities of the response function.

Conclusion—In this Letter, we have experimentally demonstrated that the ensemble distribution of every CTD quantity $(\tau_{\mathrm{WS}}, \tau_{xx}, \tau_{xy}, \tau_{\delta R}, \tau_{\delta T})$ of non-Hermitian complex scattering systems have simple asymptotic behavior with a -3 power-law tail, and that this feature is superuniversal regardless of (i) wave propagation dimension $\mathcal{D}$, (ii) number of scattering channels M, (iii) Dyson symmetry class β, and (iv) uniform absorption strength η, at least in the range that we have studied experimentally and through RMT numerics. This result is unexpected on the basis of theory for Hermitian scattering systems. Further, we have introduced the transmission time-delay difference $\tau_{\delta T}$, appropriate for nonreciprocal systems. The simple asymptotic form the distributions take implies an abundance of singular events in arbitrary non-Hermitian scattering systems, such as acoustic, optical, or photonic resonators. We suggest that the origin of the observed superuniversality is tied to the topological properties of the singularities associated with the extreme ends of the distribution.

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[1] E. P. Wigner, Phys. Rev. **98**, 145 (1955).
[2] F. T. Smith, Phys. Rev. **118**, 349 (1960).
[3] C.-H. Zhang and U. Thumm, Phys. Rev. A **84**, 033401 (2011).
[4] D. Trabert, S. Brennecke, K. Fehre, N. Anders, A. Geyer, S. Grundmann, M. S. Schöffler, L. P. H. Schmidt, T. Jahnke, R. Dörner, M. Kunitski, and S. Eckart, Nat. Commun. **12**, 1697 (2021).
[5] M. Asano, K. Y. Bliokh, Y. P. Bliokh, A. G. Kofman, R. Ikuta, T. Yamamoto, Y. S. Kivshar, L. Yang, N. Imoto, Ş. K. Özdemir, and F. Nori, Nat. Commun. **7**, 13488 (2016).
[6] R. Bourgain, J. Pellegrino, S. Jennewein, Y. R. P. Sortais, and A. Browaeys, Opt. Lett. **38**, 1963 (2013).
[7] P. C. Deshmukh, S. Banerjee, A. Mandal, and S. T. Manson, Eur. Phys. J. Special Topics **230**, 4151 (2021).
[8] J. Fuchs, N. Douguet, S. Donsa, F. Martin, J. Burgdörfer, L. Argenti, L. Cattaneo, and U. Keller, Optica **7**, 154 (2020).
[9] J. Rist *et al.*, Nat. Commun. **12**, 6657 (2021).

[10] K. I. Elghazawy and C. H. Greene, J. Phys. B **56**, 175201 (2023).
[11] A. S. Kheifets, J. Phys. B **56**, 022001 (2023).
[12] U. Saalmann and J. M. Rost, Can. J. Phys. **103**, 34 (2024).
[13] W. Jaworski and D. M. Wardlaw, Phys. Rev. A **37**, 2843 (1988).
[14] H. G. Winful, Phys. Rep. **436**, 1 (2006).
[15] S. Fan and J. M. Kahn, Opt. Lett. **30**, 135 (2005).
[16] J. Carpenter, B. J. Eggleton, and J. Schröder, Nat. Photonics **9**, 751 (2015).
[17] W. Xiong, P. Ambichl, Y. Bromberg, B. Redding, S. Rotter, and H. Cao, Phys. Rev. Lett. **117**, 053901 (2016).
[18] U. R. Patel and E. Michielssen, IEEE Trans. Antennas Propag. **69**, 902 (2021).
[19] Y. Mao, U. R. Patel, and E. Michielssen, IEEE Trans. Antennas Propag. **71**, 5266 (2023).
[20] S. Rotter, P. Ambichl, and F. Libisch, Phys. Rev. Lett. **106**, 120602 (2011).
[21] B. Gérardin, J. Laurent, P. Ambichl, C. Prada, S. Rotter, and A. Aubry, Phys. Rev. B **94**, 014209 (2016).
[22] M. Horodynski, M. Kühmayer, A. Brandstötter, K. Pichler, Y. V. Fyodorov, U. Kuhl, and S. Rotter, Nat. Photonics **14**, 149 (2020).
[23] B. W. Frazier, T. M. Antonsen, S. M. Anlage, and E. Ott, Phys. Rev. Res. **2**, 043422 (2020).
[24] P. del Hougne, K. B. Yeo, P. Besnier, and M. Davy, Laser Photonics Rev. **15**, 2000471 (2021).
[25] U. Smilansky, J. Phys. A **50**, 215301 (2017).
[26] U. Smilansky and H. Schanze, J. Phys. A **51**, 075302 (2018).
[27] P. W. Brouwer, Phys. Rev. E **68**, 046205 (2003).
[28] Y. V. Fyodorov and E. Safonova, Phys. Rev. E **108**, 044206 (2023).
[29] Y. V. Fyodorov and H.-J. Sommers, J. Math. Phys. (N.Y.) **38**, 1918 (1997).
[30] Y. V. Fyodorov, S. Suwunnarat, and T. Kottos, J. Phys. A **50**, 30LT01 (2017).
[31] Y. Fyodorov, Acta Phys. Pol. A **136**, 785 (2019).
[32] M. Osman and Y. V. Fyodorov, Phys. Rev. E **102**, 012202 (2020).
[33] L. Chen, S. M. Anlage, and Y. V. Fyodorov, Phys. Rev. E **103**, L050203 (2021).
[34] L. Chen and S. M. Anlage, Phys. Rev. E **105**, 054210 (2022).
[35] A. Z. Genack, P. Sebbah, M. Stoytchev, and B. A. van Tiggelen, Phys. Rev. Lett. **82**, 715 (1999).
[36] A. A. Chabanov, Z. Q. Zhang, and A. Z. Genack, Phys. Rev. Lett. **90**, 203903 (2003).
[37] P. Ambichl, A. Brandstötter, J. Böhm, M. Kühmayer, U. Kuhl, and S. Rotter, Phys. Rev. Lett. **119**, 033903 (2017).
[38] P. Pradhan, P. Sahay, and H. M. Almabadi, arXiv:1812.10892.
[39] Y. Huang and A. Z. Genack, in *2019 Conference on Lasers and Electro-Optics (CLEO)* (2019), p. 12.
[40] P. del Hougne, K. B. Yeo, P. Besnier, and M. Davy, Phys. Rev. Lett. **126**, 193903 (2021).
[41] U. R. Patel, Y. Mao, and E. Michielssen, J. Acoust. Soc. Am. **153**, 2769 (2023).
[42] N. Lehmann, D. Savin, V. Sokolov, and H.-J. Sommers, Physica (Amsterdam) **86D**, 572 (1995).
[43] V. A. Gopar, P. A. Mello, and M. Büttiker, Phys. Rev. Lett. **77**, 3005 (1996).
[44] T. S. Misirpashaev, P. W. Brouwer, and C. W. J. Beenakker, Phys. Rev. Lett. **79**, 1841 (1997).
[45] Y. V. Fyodorov, D. V. Savin, and H.-J. Sommers, Phys. Rev. E **55**, R4857 (1997).
[46] Y. V. Fyodorov and Y. Alhassid, Phys. Rev. A **58**, R3375 (1998).
[47] B. A. van Tiggelen, P. Sebbah, M. Stoytchev, and A. Z. Genack, Phys. Rev. E **59**, 7166 (1999).
[48] P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Waves Random Media **9**, 91 (1999).
[49] D. V. Savin, Y. V. Fyodorov, and H.-J. Sommers, Phys. Rev. E **63**, 035202(R) (2001).
[50] T. Kottos and U. Smilansky, J. Phys. A **36**, 3501 (2003).
[51] F. Mezzadri and N. J. Simm, Commun. Math. Phys. **324**, 465 (2013).
[52] C. Texier and S. N. Majumdar, Phys. Rev. Lett. **110**, 250602 (2013).
[53] M. Novaes, J. Math. Phys. (N.Y.) **56**, 062110 (2015).
[54] F. D. Cunden, Phys. Rev. E **91**, 060102(R) (2015).
[55] Y. Huang, C. Tian, V. A. Gopar, P. Fang, and A. Z. Genack, Phys. Rev. Lett. **124**, 057401 (2020).
[56] P. Šeba, K. Życzkowski, and J. Zakrzewski, Phys. Rev. E **54**, 2438 (1996).
[57] Y. V. Fyodorov and H.-J. Sommers, Phys. Rev. Lett. **76**, 4709 (1996).
[58] P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Phys. Rev. Lett. **78**, 4737 (1997).
[59] D. V. Savin and H.-J. Sommers, Phys. Rev. E **68**, 036211 (2003).
[60] A. Ossipov and Y. V. Fyodorov, Phys. Rev. B **71**, 125133 (2005).
[61] T. Kottos, J. Phys. A **38**, 10761 (2005).
[62] A. Grabsch, D. V. Savin, and C. Texier, J. Phys. A **51**, 404001 (2018).
[63] A. Grabsch, J. Phys. A **53**, 025202 (2019).
[64] F. J. Dyson, J. Math. Phys. (N.Y.) **3**, 1199 (1962).
[65] C. Texier, Physica (Amsterdam) **82E**, 16 (2016).
[66] Y. V. Fyodorov and D. V. Savin, J. Exp. Theor. Phys. Lett. **80**, 725 (2004).
[67] D. V. Savin, H.-J. Sommers, and Y. V. Fyodorov, J. Exp. Theor. Phys. Lett. **82**, 544 (2005).
[68] Y. V. Fyodorov, D. V. Savin, and H.-J. Sommers, J. Phys. A **38**, 10731 (2005).
[69] T. M. A. Xing Zheng and E. Ott, Electromagnetics **26**, 37 (2006).
[70] T. M. A. Xing Zheng and E. Ott, Electromagnetics **26**, 3 (2006).
[71] X. Zheng, S. Hemmady, T. M. Antonsen, S. M. Anlage, and E. Ott, Phys. Rev. E **73**, 046208 (2006).
[72] S. Hemmady, J. Hart, X. Zheng, T. M. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. B **74**, 195326 (2006).
[73] S. Hemmady, X. Zheng, T. M. Antonsen, E. Ott, and S. M. Anlage, Acta Phys. Pol. A **109**, 65 (2006).
[74] S. Hemmady, X. Zheng, J. Hart, T. M. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E **74**, 036213 (2006).

[75] J.-H. Yeh, Z. Drikas, J. G. Gil, S. Hong, B. Taddese, E. Ott, T. Antonsen, T. Andreadis, and S. Anlage, Acta Phys. Pol. A **124**, 1045 (2013).
[76] C. Beenakker and P. Brouwer, Physica E **9**, 463 (2001).
[77] Y. V. Fyodorov, J. Exp. Theor. Phys. Lett. **78**, 250 (2003).
[78] M. Novaes and J. Kuipers, Phys. Rev. E **108**, 034201 (2023).
[79] M. Novaes, J. Stat. Phys. **190**, 182 (2023).
[80] J. Böhm, A. Brandstötter, P. Ambichl, S. Rotter, and U. Kuhl, Phys. Rev. A **97**, 021801(R) (2018).
[81] P. Bredol, Phys. Rev. B **103**, 035404 (2021).
[82] Y. D. Chong, L. Ge, H. Cao, and A. D. Stone, Phys. Rev. Lett. **105**, 053901 (2010).
[83] L. Chen, T. Kottos, and S. M. Anlage, Nat. Commun. **11**, 5826 (2020).
[84] M. F. Imani, D. R. Smith, and P. del Hougne, Adv. Funct. Mater. **30**, 2005310 (2020).
[85] J. Erb, D. Shrekenhamer, T. Sleasman, T. Antonsen, and S. Anlage, Acta Phys. Pol. A **144**, 421 (2024).
[86] F. T. Faul, L. Cronier, A. Alhulaymi, A. D. Stone, and P. del Hougne, arXiv:2407.00054.
[87] S. Dutta-Gupta, O. J. F. Martin, S. D. Gupta, and G. S. Agarwal, Opt. Express **20**, 1330 (2012).
[88] K. Pichler, M. Kühmayer, J. Böhm, A. Brandstötter, P. Ambichl, U. Kuhl, and S. Rotter, Nature (London) **567**, 351 (2019).
[89] D. G. Baranov, A. Krasnok, T. Shegai, A. Alù, and Y. Chong, Nat. Rev. Mater. **2**, 17064 (2017).
[90] J. Z. Song, P. Bai, Z. H. Hang, and Y. Lai, New J. Phys. **16**, 033026 (2014).
[91] C. Meng, X. Zhang, S. T. Tang, M. Yang, and Z. Yang, Sci. Rep. **7**, 43574 (2017).
[92] Y. Li, M. Qi, J. Li, P.-C. Cao, D. Wang, X.-F. Zhu, C.-W. Qiu, and H. Chen, Nat. Commun. **13**, 2683 (2022).
[93] A. N. Vetlugin, Phys. Rev. A **104**, 013716 (2021).
[94] A. N. Vetlugin, F. Martinelli, S. Dong, and C. Soci, Nanophotonics **12**, 505 (2023).
[95] A. N. Vetlugin, R. Guo, C. Soci, and N. I. Zheludev, New J. Phys. **24**, 122001 (2022).
[96] Y. Huang, Y. Kang, and A. Z. Genack, Phys. Rev. Res. **4**, 013102 (2022).
[97] J. Sol, A. Alhulaymi, A. D. Stone, and P. del Hougne, Sci. Adv. **9**, eadf0323 (2023).
[98] X. Jiang, S. Yin, H. Li, J. Quan, H. Goh, M. Cotrufo, J. Kullig, J. Wiersig, and A. Alù, Nat. Phys. **20**, 109 (2024).
[99] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, Phys. Rev. Lett. **106**, 213901 (2011).
[100] C. Kurter, P. Tassin, L. Zhang, T. Koschny, A. P. Zhuravel, A. V. Ustinov, S. M. Anlage, and C. M. Soukoulis, Phys. Rev. Lett. **107**, 043901 (2011).
[101] B. Peng, Ş. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. **10**, 394 (2014).
[102] L. Chen, S. M. Anlage, and Y. V. Fyodorov, Phys. Rev. Lett. **127**, 204101 (2021).
[103] J. Verbaarschot, H. Weidenmüller, and M. Zirnbauer, Phys. Rep. **129**, 367 (1985).
[104] V. Sokolov and V. Zelevinsky, Nucl. Phys. **A504**, 562 (1989).
[105] Y. V. Fyodorov and D. V. Savin, in *The Oxford Handbook of Random Matrix Theory*, edited by G. Akemann, J. Baik, and P. D. Francesco (Oxford University Press, New York, 2011), pp. 703–722.
[106] U. Kuhl, O. Legrand, and F. Mortessagne, Fortschr. Phys. **61**, 404 (2013).
[107] H. Schomerus, in *Stochastic Processes and Random Matrices: Lecture Notes of the Les Houches Summer School 2015*, edited by G. Schehr, A. Altland, Y. V. Fyodorov, N. O'Connell, and L. F. Cugliandolo (Oxford University Press, New York, 2017), pp. 409–473.
[108] Y. Kang and A. Z. Genack, Phys. Rev. B **103**, L100201 (2021).
[109] D. S. Fisher and P. A. Lee, Phys. Rev. B **23**, 6851 (1981).
[110] S. Rotter and S. Gigan, Rev. Mod. Phys. **89**, 015005 (2017).
[111] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).
[112] P. W. Brouwer and C. W. J. Beenakker, Phys. Rev. B **55**, 4695 (1997).
[113] S. Ma and S. M. Anlage, Adv. Opt. Mater. **12**, 2301852 (2023).
[114] P. Sebbah, O. Legrand, and A. Z. Genack, Phys. Rev. E **59**, 2406 (1999).
[115] K. J. H. van Bemmel, H. Schomerus, and C. W. J. Beenakker, Phys. Scr. **2001**, 278 (2001).
[116] M. Białous, P. Dulian, A. Sawicki, and L. Sirko, Phys. Rev. E **104**, 024223 (2021).
[117] A. Z. Genack, Y. Huang, A. Maor, and Z. Shi, Nat. Commun. **15**, 2606 (2024).
[118] L. Chen, I. L. Giovannelli, N. Shaibe, and S. M. Anlage, Phys. Rev. B **110**, 045103 (2024).
[119] Y. Kang and A. Z. Genack, Phys. Rev. B **103**, L100201 (2021).
[120] P. So, S. M. Anlage, E. Ott, and R. N. Oerter, Phys. Rev. Lett. **74**, 2662 (1995).
[121] U. Stoffregen, J. Stein, H.-J. Stöckmann, M. Kuś, and F. Haake, Phys. Rev. Lett. **74**, 2666 (1995).
[122] D. H. Wu, J. S. A. Bridgewater, A. Gokirmak, and S. M. Anlage, Phys. Rev. Lett. **81**, 2890 (1998).
[123] S. H. Chung, A. Gokirmak, D. H. Wu, J. S. A. Bridgewater, E. Ott, T. M. Antonsen, and S. M. Anlage, Phys. Rev. Lett. **85**, 2482 (2000).
[124] H. Schanze, H.-J. Stöckmann, M. Martínez-Mares, and C. H. Lewenkopf, Phys. Rev. E **71**, 016223 (2005).
[125] M. Ławniczak, S. Bauch, O. Hul, and L. Sirko, Phys. Scr. **2009**, 014050 (2009).
[126] M. Ławniczak, S. Bauch, O. Hul, and L. Sirko, Phys. Rev. E **81**, 046204 (2010).
[127] M. Ławniczak, S. Bauch, O. Hul, and L. Sirko, Phys. Scr. **2011**, 014014 (2011).
[128] M. Ławniczak and L. Sirko, Sci. Rep. **9**, 5630 (2019).
[129] X. Zhang, W. Zhang, J. Che, and B. Dietz, Phys. Rev. E **108**, 044211 (2023).
[130] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.134.147203 for further description of the experimental systems, RMT numerical results, and brief discussion of the limits of the superuniversal statistics, which includes Ref. [131].
[131] M.-J. Lee, Statistical modeling of wave chaotic transport and tunneling, Phd thesis, University of Maryland, College Park, MD, 2013, available at xhttp://hdl.handle.net/1903/14524.

[132] E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990).
[133] H.-J. Stöckmann and J. Stein, Phys. Rev. Lett. **64**, 2215 (1990).
[134] H.-J. Stöckmann, *Quantum Chaos An Introduction* (Cambridge University Press, Cambridge, England, 1999).
[135] A. Richter, Phys. Scr. **2001**, 212 (2001).
[136] O. Hul, M. Ławniczak, S. Bauch, A. Sawicki, M. Kuś, and L. Sirko, Phys. Rev. Lett. **109**, 040402 (2012).
[137] G. Gradoni, J.-H. Yeh, B. Xiao, T. M. Antonsen, S. M. Anlage, and E. Ott, Wave Motion **51**, 606 (2014).
[138] B. Dietz and A. Richter, Chaos **25**, 097601 (2015).
[139] S. Hemmady, X. Zheng, E. Ott, T. M. Antonsen, and S. M. Anlage, Phys. Rev. Lett. **94**, 014102 (2005).
[140] J. A. Hart, T. M. Antonsen, and E. Ott, Phys. Rev. E **80**, 041109 (2009).
[141] J.-H. Yeh, J. A. Hart, E. Bradshaw, T. M. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E **82**, 041114 (2010).
[142] M. Ławniczak, S. Bauch, O. Hul, and L. Sirko, Phys. Scr. **2012**, 014018 (2012).
[143] S. Hemmady, T. M. Antonsen, E. Ott, and S. M. Anlage, IEEE Transactions on Electromagnetic Compatibility **54**, 758 (2012).
[144] M.-J. Lee, T. M. Antonsen, and E. Ott, Phys. Rev. E **87**, 062906 (2013).
[145] Y. Aurégan and V. Pagneux, Acta Acust. United Acust. **102**, 869 (2016).
[146] B. Dietz, V. Yunko, M. Białous, S. Bauch, M. Ławniczak, and L. Sirko, Phys. Rev. E **95**, 052202 (2017).
[147] Z. Fu, T. Koch, T. Antonsen, and S. Anlage, Acta Phys. Pol. A **132**, 1655 (2017).
[148] V. L. Lyuboshitz, Phys. Lett. B **72**, 41 (1977).
[149] R. Pierrat, P. Ambichl, S. Gigan, A. Haber, R. Carminati, and S. Rotter, Proc. Natl. Acad. Sci. U.S.A. **111**, 17765 (2014).
[150] T. Kottos and U. Smilansky, Phys. Rev. Lett. **79**, 4794 (1997).
[151] O. Hul, S. Bauch, P. Pakoński, N. Savytskyy, K. Życzkowski, and L. Sirko, Phys. Rev. E **69**, 056205 (2004).
[152] H. Li, S. Suwunnarat, R. Fleischmann, H. Schanz, and T. Kottos, Phys. Rev. Lett. **118**, 044101 (2017).
[153] T. Hofmann, J. Lu, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. E **104**, 045211 (2021).
[154] J. Lu, T. Hofmann, U. Kuhl, and H.-J. Stöckmann, Entropy **25**, 109 (2023).
[155] T. Ghutishvili, L. Chen, S. M. Anlage, and T. M. Antonsen, Phys. Rev. Res. **5**, 033195 (2023).
[156] B. Dietz, T. Klaus, M. Masi, M. Miski-Oglu, A. Richter, T. Skipa, and M. Wunderle, Phys. Rev. E **109**, 034201 (2024).
[157] A. Rehemanjiang, M. Richter, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. E **97**, 022204 (2018).
[158] J. Che, J. Lu, X. Zhang, B. Dietz, and G. Chai, Phys. Rev. E **103**, 042212 (2021).
[159] J. Che, X. Zhang, W. Zhang, B. Dietz, and G. Chai, Phys. Rev. E **106**, 014211 (2022).
[160] M. Ławniczak, A. Akhshani, O. Farooq, M. Białous, S. Bauch, B. Dietz, and L. Sirko, Phys. Rev. E **107**, 024203 (2023).
[161] H. Alt, C. I. Barbosa, H.-D. Gräf, T. Guhr, H. L. Harney, R. Hofferbert, H. Rehfeld, and A. Richter, Phys. Rev. Lett. **81**, 4847 (1998).
[162] B. Dietz, T. Guhr, H. L. Harney, and A. Richter, Phys. Rev. Lett. **96**, 254101 (2006).
[163] S. Deus, P. M. Koch, and L. Sirko, Phys. Rev. E **52**, 1146 (1995).
[164] H. Alt, C. Dembowski, H. D. Graf, R. Hofferbert, H. Rehfeld, A. Richter, R. Schuhmann, and T. Weiland, Phys. Rev. Lett. **79**, 1026 (1997).
[165] G. B. Tait, R. E. Richardson, M. B. Slocum, and M. O. Hatfield, IEEE Transactions on Electromagnetic Compatibility **53**, 846 (2011).
[166] B. Xiao, T. M. Antonsen, E. Ott, Z. B. Drikas, J. G. Gil, and S. M. Anlage, Phys. Rev. E **97**, 062220 (2018).
[167] P. del Hougne, M. Davy, and U. Kuhl, Phys. Rev. Appl. **13**, 041004(R) (2020).
[168] J.-B. Gros, P. del Hougne, and G. Lerosey, Phys. Rev. A **101**, 061801(R) (2020).
[169] B. W. Frazier, T. M. Antonsen, S. M. Anlage, and E. Ott, Phys. Rev. Appl. **17**, 024027 (2022).
[170] H.-T. Chen, A. J. Taylor, and N. Yu, Rep. Prog. Phys. **79**, 076401 (2016).
[171] M. Elsawy, C. Kyrou, E. Mikheeva, R. Colom, J.-Y. Duboz, K. Z. Kamali, S. Lanteri, D. Neshev, and P. Genevet, Laser Photonics Rev. **17**, 2200880 (2023).
[172] T. Sleasman, R. Duggan, R. S. Awadallah, and D. Shrekenhamer, Phys. Rev. Appl. **20**, 014004 (2023).
[173] B. Dietz, T. Friedrich, H. L. Harney, M. Miski-Oglu, A. Richter, F. Schäfer, and H. A. Weidenmüller, Phys. Rev. Lett. **98**, 074103 (2007).
[174] B. Dietz, T. Friedrich, H. L. Harney, M. Miski-Oglu, A. Richter, F. Schäfer, J. Verbaarschot, and H. A. Weidenmüller, Phys. Rev. Lett. **103**, 064101 (2009).
[175] S. Bittner, B. Dietz, H. L. Harney, M. Miski-Oglu, A. Richter, and F. Schäfer, Phys. Rev. E **89**, 032909 (2014).
[176] F. Castañeda Ramírez, A. M. Martínez-Argüello, T. Hofmann, A. Rehemanjiang, M. Martínez-Mares, J. A. Méndez-Bermúdez, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. E **105**, 014202 (2022).
[177] S. Hemmady, X. Zheng, T. M. Antonsen, E. Ott, and S. M. Anlage, Phys. Rev. E **71**, 056215 (2005).
[178] O. Hul, O. Tymoshchuk, S. Bauch, P. M. Koch, and L. Sirko, J. Phys. A **38**, 10489 (2005).
[179] R. A. Méndez-Sánchez, U. Kuhl, M. Barth, C. H. Lewenkopf, and H.-J. Stöckmann, Phys. Rev. Lett. **91**, 174102 (2003).
[180] M. Ławniczak, O. Hul, S. Bauch, P. Seba, and L. Sirko, Phys. Rev. E **77**, 056210 (2008).
[181] J. Kuipers, D. V. Savin, and M. Sieber, New J. Phys. **16**, 123018 (2014).
[182] J. Erb, N. Shaibe, R. Calvo, D. Lathrop, T. Antonsen, T. Kottos, and S. M. Anlage, arXiv:2411.01069.
[183] M. Stein, S. Keller, Y. Luo, and O. Ilic, Nat. Commun. **13**, 6533 (2022).
[184] L. Huang, S. Huang, C. Shen, S. Yves, A. S. Pilipchuk, X. Ni, S. Kim, Y. K. Chiang, D. A. Powell, J. Zhu, Y. Cheng, Y. Li, A. F. Sadreev, A. Alù, and A. E. Miroshnichenko, Nat. Rev. Phys. **6**, 11 (2024).

[185] S. A. Cummer, J. Christensen, and A. Alù, Nat. Rev. Mater. **1**, 16001 (2016).

[186] W. Wan, Y. Chong, L. Ge, H. Noh, A. D. Stone, and H. Cao, Science **331**, 889 (2011).

[187] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

[188] D. R. Solli, C. F. McCormick, R. Y. Chiao, S. Popescu, and J. M. Hickmann, Phys. Rev. Lett. **92**, 043601 (2004).

[189] M. R. Dennis and J. B. Götte, New J. Phys. **14**, 073013 (2012).

[190] Y. Huang, Phys. Rev. Res. **4**, 013102 (2022): "The distribution of transmission zeros is expected to differ from the distributions for zeros of the scattering and reflection matrices since the transmission zeros do not correspond to eigenvalues of an effective internal Hamiltonian".

[191] M. V. Berry and M. R. Dennis, Proc. R. Soc. A **456**, 2059 (2000).

[192] J. C. Neu, Physica (Amsterdam) **43D**, 385 (1990).