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Why can't experimentalists agree on the superconducting critical exponents?

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Abstract

The scaling analysis of voltage vs. current curves has been an invaluable tool in the study of the normal-superconducting phase transition, both in zero-field and in the vortex–glass transition in a field. However, we have recently shown that the conventional scaling analysis is too flexible to uniquely determine the critical parameters. We have also shown that extrinsic effects such as current noise, small magnetic fields (for the zero-field transition), and finite size effects can obscure and even destroy the three-dimensional phase transition. These factors have led to the wide range of values for the dynamic critical exponent z and the static critical exponent ν reported in the literature, even for the zero-field transition. We have developed a criterion that removes the flexibility in the scaling analysis and have conducted experiments to eliminate the extrinsic effects described above. Our results show that finite size effects, which obscure the phase transition in thin films, are absent in bulk, untwinned, single crystals.
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The experimentally-accessible critical regimes of the cuprate superconductors [1] excited a great interest in experiments on the normal-superconducting phase transition in these materials. Scaling theory predicts that the phase transition is governed by the static and dynamic critical exponents ν and z, respectively [2]. This theory is valid for the vortex–glass transition in a field and the zero-field transition (though values of ν and z may differ). In zero-field, the transition is expected to obey the three-dimensional (3D)-XY theory, where $\nu \approx 0.67$ and $z = 2.0$ (assuming diffusive dynamics).

In the past two decades, there have been numerous experiments which have probed this phase transition. In zero-field, measurements of specific heat [3], penetration depth [4], and thermal expansivity [5] on bulk single crystals have led to a consensus that the phase transition does follow the 3D-XY theory where $\nu \approx 0.67$ (although the value of z is unaddressed). There are even some transport measurements on crystals [6] that find a similar 3D-XY static exponent.

By far the most popular tool to examine the normal-superconducting phase transition is the measurement of current vs. voltage (I–V) curves on thin films (thickness 1000 Å $\lesssim d \lesssim$ 4000 Å). However, I–V curves in zero or low magnetic fields yield exponents ranging from $\nu \approx 0.63$ and $z \approx 1.25$ [7] to as high as $\nu \approx 1.1$ and $z \approx 8.3$ [8]. For the vortex–glass transition, the range of exponents is even wider [9].

Our own efforts to understand this complex topic began with a re-examination of the vortex–glass transition. We

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use thin films of optimally doped $YBa_2Cu_3O_{7-\delta}$ grown via pulsed laser deposition onto $SrTiO_3$ substrates. X-ray diffraction shows our films to be predominately *c*-axis oriented with transition widths $\leqslant 0.25$ K measured via ac susceptibility. $R(T)$ measurements show $T_c \approx 91.5$ K and $\Delta T_c \approx 0.7$ K. These films are patterned into bridges 8 μm by 40 μm and etched using dilute phosphoric acid. Typical *I–V* curves taken on 2200 Å thick film in a field of 4 T are shown in Fig. 1a.

Scaling predicts [2] $V\xi^{2+z-D}/I = \chi \pm (I\ \xi^{D-1}/T)$, where D is the dimensionality, z is the dynamic critical exponent, and $\chi_\pm$ are the scaling function above and below the transition temperature T_g. ξ is the correlation length, expected to go as $\xi \sim |(T - T_g)/T_g|^{-\nu}$, where ν is the static exponent. In zero-field, replace T_g, the glass transition temperature, with T_c, the zero-field transition temperature. The scaling equation has two useful limits. At the transition temperature, $V \sim I^{(z+1)/2}$ (for 3D), in other words, a power-law for all currents. For $T > T_g$, as $I \rightarrow 0$, we expect an ohmic response that varies with temperature as $V/I \sim |T - T_g|^{\nu(z-1)}$.

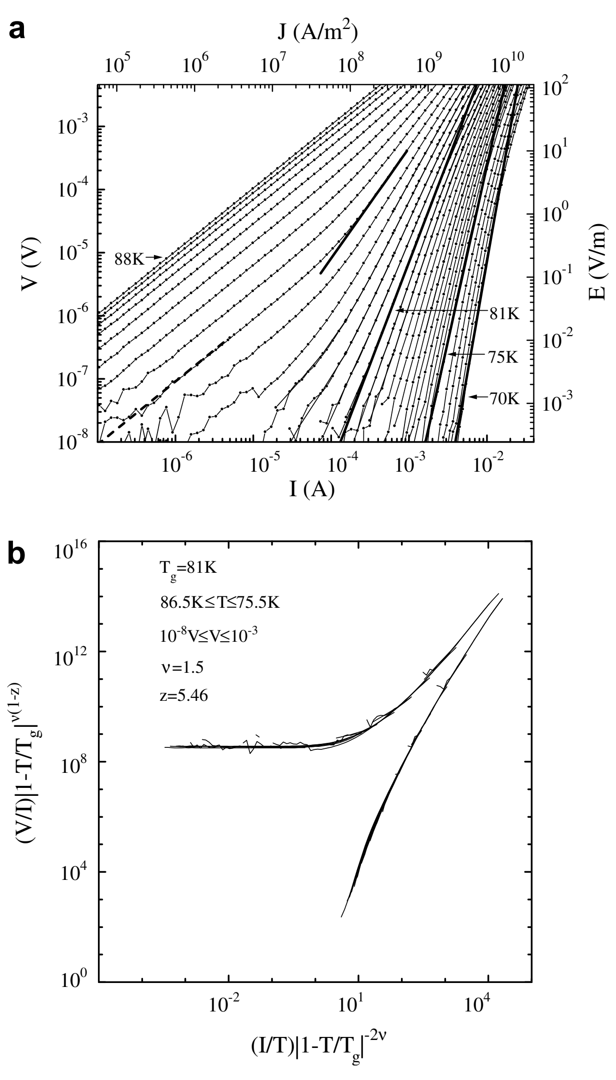

Fig. 1. Panel (a) shows *I–V* curves for a 2200 Å thick film with bridge dimensions 8 μm ×40 μm in a field of 4 T. The dashed line has a slope of one. The solid lines are power-law fits to non-ohmic isotherms. The conventional data collapse for $T_g = 81$ K is shown in (b). From Ref. [10].

These equations predict ohmic behavior above T_g. Following the conventional analysis, we return to Fig. 1a and choose the first isotherm without an ohmic "tail" at low currents to be T_g, so $T_g = 81$ K. A power-law fit of the isotherm will allow us to determine the exponent z (using $V \sim I^{(z+1)/2}$). Once we know z, we can determine ν from the ohmic tails above the critical isotherm (using $V/I \sim |T - T_g|^{\nu(z-1)}$). Finally, knowing z and ν, we can collapse the data. If we have correctly determined the critical exponents, then the data should fall on two universal curves, $\chi_\pm$, for above and below the critical temperature. This data collapse is shown in Fig. 1b.

A closer look at the isotherms in Fig. 1a suggests that the standard procedure of choosing the critical isotherm – the first isotherm without an ohmic tail – is flawed. If our voltmeter measured only to the microvolt range, then the first isotherm without a measurable ohmic tail would be above 81 K. Similarly, our voltage resolution in the nanovolt range leads us to wonder whether the 81 K isotherm has an ohmic tail *below* the resolution of our voltmeter, and perhaps the true transition temperature lies lower than 81 K. To test this conjecture, we can choose other isotherms (e.g. 75 and 70 K) as the transition temperature and repeat the conventional analysis. This creates different sets of T_g, ν, and z. With each set of critical parameters, we can continue the conventional analysis, and for each set we can create a data collapse – each of which looks as good to the eye as the collapse shown in Fig. 1b [10]. Clearly, the conventional data analysis and its accompanying data collapse *cannot* uniquely determine the critical parameters.

To resolve this, we propose the *opposite concavity criterion* [10]: isotherms equidistant from the critical isotherm should have opposite concavities. This criterion is easiest to see on a plot of $d\,\mathrm{Log}(V)/d\,\mathrm{Log}(I)$ vs. I, where isotherms above and below the transition have opposite slopes and the critical isotherm is a horizontal line whose intercept is $(z+1)/2$. We show two logarithmic derivative plots for isotherms taken in a field of 4 T and in zero-field in Fig. 2.

In Fig. 2a, there is no isotherm that is horizontal over the entire range of currents in a field, indicating that the opposite concavity criterion is not obeyed. Moreover, all the isotherms bend towards $d\,\mathrm{Log}(V)/d\,\mathrm{Log}(I) = 1$ (ohmic behavior), and isotherms at low temperatures without ohmic tails are merely limited by the sensitivity of our voltmeter. These results imply that there is no phase transition in a magnetic field. More shocking is that Fig. 2b gives a similar result for zero-field, implying that there is no transition to a zero-resistance state, even in zero-field! These surprising results lead us to question whether the hallmark of isotherms greater than the critical temperature – the low-current ohmic tail – could be created by intrinsic or extrinsic effects other than the phase transition.

Our first realization was that current noise can create a linear response in non-linear devices. For most applica-

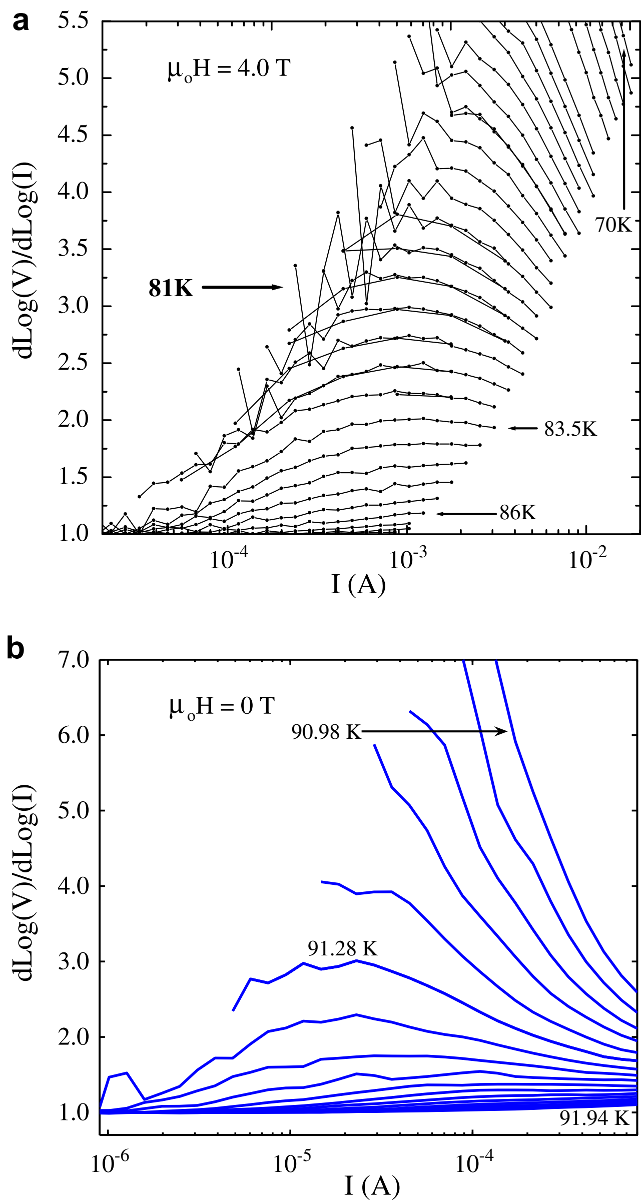

Fig. 2. Logarithmic derivatives for two similar ($d \approx 2200$ Å) films in (a) $\mu_o H = 4$ T and (b) $\mu_o H = 0$ T. The opposite concavity criterion cannot be seen, and all isotherms (to the limit of our voltage sensitivity) bend back towards $\mathrm{dLog}(V)/\mathrm{dLog}(I) = 1$, or ohmic behavior. This implies that no phase transition exists, either in field or in zero-field. From Refs. [10,12].

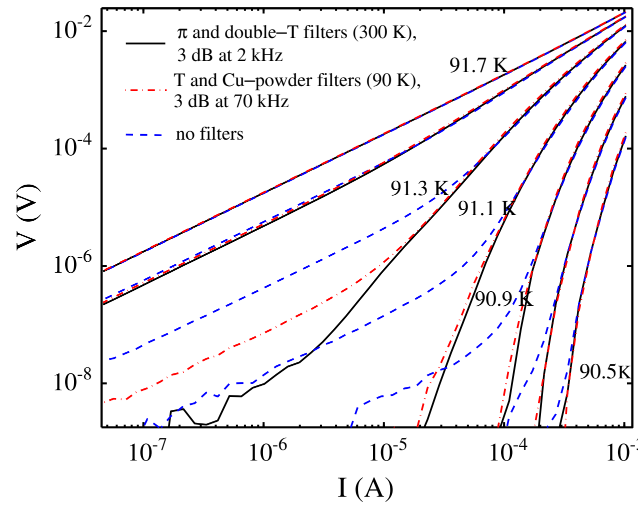

Fig. 3. I–V curves for a 2100 Å thick film showing the effects of current noise. Low-current ohmic tails are created in non-linear isotherms, masking the normal-superconducting phase transition. From Ref. [11].

tions, the applied current I is not a delta function but rather a probability distribution centered about I with some width σ_I. For high currents, when $I \gg \sigma_I$, it is easy to show that the measured voltage is $\langle V \rangle \approx f(I)$, where $f(I)$ is the true non-linear response of the system. However, for $I \ll \sigma_I$, $\langle V \rangle \approx IR_{\mathrm{eff}}$, where R_{eff} is an effective resistance that depends on σ_I [11]. The result is ohmic response at low currents, mimicking exactly the hallmark of the phase transition.

Our cryostat is filtered using π and double-T section filters, creating a 3 dB point at 3 kHz. We can change the current noise in the sample by removing the filters, as shown in Fig. 3. In this figure, the isotherm at 91.1 K changes from non-linear to linear behavior as we increase the current noise. Thus, if current leads are not filtered, it will be impossible to measure the true normal-superconducting phase transition. This is true in zero-field as well as for the vortex–glass transition [11].

Second, we found that even very small magnetic fields can obscure the zero-field phase transition [13]. Just 1 mT is large enough to create ohmic behavior in the non-linear isotherms, and fields as small as the Earth's ambient field (50 μT) can change the shape of the I–V curves. These effects can be eliminated by shielding the cryostat with μ-metal shields, which reduce the field at the sample to 0.2 μT. Because researchers often measure "zero" field data in the Earth's ambient field or inside the remnant field of a superconducting magnet, improper shielding can help explain the wide variety of exponents found in zero-field.

Finally, we found that finite size effects seen in thinner films [14,6] occur also in thick films [12]. Different current densities probe fluctuations of different sizes, given by $J \sim T\xi^{1-D}$. This means that as J decreases, the size of the fluctuation increases until the current probes only fluctuations limited by the thickness of the film. This occurs at a current density given by

$$J_{\mathrm{cross}} = ck_{\mathrm{B}}T/\Phi_{\mathrm{o}}d^2$$

[2,15], where k_{B} is the Boltzmann constant, Φ_{o} is the magnetic flux quantum, d is the thickness of the film, and c is a constant of order 1 [2,14]. In a bridge of dimensions 8×40 μm^2 that is 2200 Å thick, this crossover is expected to occur at about 10 μA. For $J < J_{\mathrm{cross}}$, we expect ohmic behavior as the 3D transition is limited by the thickness of the film. This is what causes the downturn in the derivative plot in Fig. 2b at $I \approx 20$ μA. We have verified this equation experimentally in films from 1000 to 3200 Å, and we find $c \approx 0.6$, in agreement with theory [12].

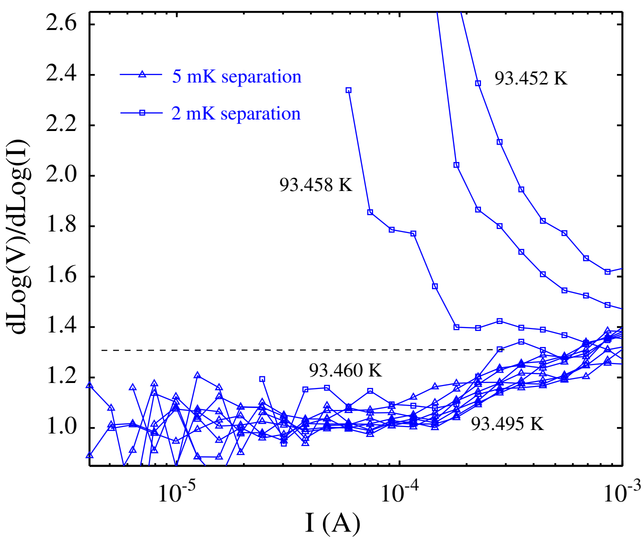

Fig. 4. Logarithmic derivatives of I–V curves for a $500 \times 330 \times 100\ \mu m^3$ single crystal. As expected, the isotherms for the crystal do not bend down towards ohmic behavior due to finite size effects, and they agree with the opposite concavity criterion. The dotted line is a horizontal line which separates isotherms above T_c from isotherms below T_c.

The most compelling evidence of the finite size effects in films is an examination of the I–V curves in high-quality bulk single crystals. The derivative plot of I–V curves taken on a crystal ($500 \times 330 \times 100\ \mu m^3$) in zero-field ($T_c \approx 93.5$ K, $\Delta T_c \approx 10$ mK) [16] are shown in Fig. 4. Here the the data agree with the opposite concavity criterion, and there is no downturn towards $d\,\mathrm{Log}(V)/d\,\mathrm{Log}(I) = 1$, as the crossover to 2D behavior occurs at currents much smaller than 1 nA. It is interesting to note that the opposite concavity criterion is also obeyed if we examine only the high-current region of thin films, where isotherms have opposite slopes for current densities greater than J_{cross}.

In conclusion, we have demonstrated that the conventional scaling analysis and data collapse of the normal-superconducting phase transition is too flexible to uniquely determine the critical exponents and critical temperature, in zero-field and in the vortex–glass transition. Moreover, extrinsic effects like current noise and small magnetic fields (for the zero-field transition) can obscure or mimic the phase transition. We have also shown that even thick films ($d \approx 3500$ Å) have finite size effects which limit the 3D phase transition to high currents in films. This analysis can also be extended to finite-frequency effects for ac measurements. The combination of these unexpected difficulties in the analysis and the experiment may well be the cause of the lack of consensus regarding the value for the critical exponents measured via I–V curves.

Finally, we have demonstrated that bulk single crystals are immune to finite size effects, as expected, and obey the opposite concavity criterion, also seen in the high-current regime in thin films. Work is ongoing to determine the value of the critical exponents ν and z in both crystals and thin films.

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