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Noise Measurements

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Abstract

- Primary noise standards
- How to make noise measurements
- Amplifier (complex) noise parameters
- Issues related to uncertainty and accuracy

The objective of the talk will be to consider how to make noise measurements, to show how the noise measurements made by end users can be traced back to primary standards and the accuracy limitations which result from this and to discuss noise parameter measurement. We will also consider ways of improving the accuracy of typical measurements.

About the Speaker

David Adamson works as lead scientist in the fields of communications metrology, THz metrology and Guided Wave metrology (focussing on power and noise in particular).

He was previously Team Manager for the RF & Microwave Team at NPL, responsible for the smooth running of all aspects of the Team. This role included responsibility for the work of the team in all areas of RF & Microwave standards work including antenna parameters, other free field parameters, dielectric measurements, and guided wave parameters including noise, power, impedance and attenuation. The RF & Microwave group at NPL is one of the world's leading groups in the field of RF & Microwave measurement standards and has an extensive area of research activity.

NOISE MEASUREMENTS

David Adamson

1 Introduction

In any classical system (ie non-quantum) the ultimate limit of sensitivity will be set either by interference or by random signals which are produced within the system. In this document we are not concerned with the situations where interference sets the ultimate limit and so henceforth, we will consider only situations where the ultimate sensitivity is set by random signals. The minimum possible value for these random signals is generally set by physical phenomena collectively called noise. If we are interested in determining the limit of sensitivity of a system then we will want to measure the random signals which determine that limit. Alternatively, we may wish to design a system to reach a chosen level of sensitivity in which case we will want to have methods to allow the calculation of the level of random signals which are to be anticipated in the system.

In this document, we are particularly interested in systems which are sensitive to electromagnetic signals, generally in the microwave and RF region of the spectrum. However, some of the principles apply at any frequency, or even to systems which are not concerned with electromagnetic signals.

Random signals produced in an electrical system are usually called electrical noise. The concept of noise is familiar to anyone who has tuned an AM radio to a point between stations where the loudspeaker will produce a hissing noise which is attributable to the electrical noise in the system. This example also illustrates an important general point about noise – the source of the noise may be either internal (caused by phenomena in the receiver in this case) or external (atmospheric and other sky noise in this case). Usually we can attempt to choose our system components to bring the noise internal to the system to a level which is appropriate for that system whereas the external noise is often fixed by other phenomena and its level can only be controlled by careful design. For a system with no antenna, careful screening may ensure that the external noise is zero but a system with an antenna will always be susceptible to some external noise and the level can only be altered by careful design and even then, only within certain limits.

Sources of internal noise include various random fluctuations of electrons in the materials making up the electrical circuits. It is important to realise that in a classical system the level of these fluctuations cannot ever be zero except when the system is entirely at a temperature of absolute zero, 0 K. There are various ways of reducing the noise –

choice of components and the temperature of the system are examples. In this document we are interested in methods of measuring the noise in an electrical system in the RF & Microwave frequency range.

It is worth spending a little time considering what sort of random signal we are thinking about when we refer to noise. A noise signal will have an arbitrary amplitude at any instant and the amplitude at another instant cannot be predicted by use of any historical data about previous amplitudes. Since noise is a random signal the time averaged offset value will be zero and consequently we consider root mean square magnitude values. The root mean square value of the voltage is proportional to the average power of the noise signal. For theoretical reasons, it is often sensible to consider the amplitude of the signal as having a Gaussian probability density function. This is because one of the major sources of noise (thermal noise) gives a theoretical Gaussian distribution and because, if the noise has a Gaussian distribution, some analyses of noise are facilitated. In a practical situation, noise is unlikely to be truly Gaussian since there will be amplitude and bandwidth limitations which will prevent this occurring. However, in a large majority of cases the assumption of a Gaussian distribution is sufficiently close to reality to make it a very satisfactory model. The term “white noise” is often used by analogy with white light to describe a situation where the noise signal covers a very large (effectively infinite) bandwidth. Of course, white light from the sun is a noise signal in the optical band.

In almost all cases there is no correlation between sources of noise and so the noise is non coherent. This means that, if we have several sources of noise in a system, the total noise can be found by summing the individual noise powers. In some cases there can be correlation between noise signals and in this case the analysis is more complex. A common example of this situation is where a noise signal generated within a system travels both towards the input and the output. If some of the noise signal is then reflected back towards the output from the input – due, for example, to a mismatch – there will be some degree of correlation between the reflected and original signals at the output.

2 Types of Noise.

2.1 Thermal Noise.

Thermal or Johnson [1] noise is the most fundamental source of noise and it is present in all

systems. At any temperature above absolute zero, the electrons (and other charges) in the materials of the circuit will have a random motion caused by the temperature. This will occur in both active and passive components of the circuit. Any movement of charges gives rise to a current and, in the presence of resistance, a voltage. The voltage will vary randomly in time and is described in terms of its mean square value. This was first done by Nyquist [2]

$$\overline{v^2} = 4kTBR \quad \text{Equation 1}$$

where:

k = Boltzmann's constant (1.38×10^{-23} joules/K)
 R = Resistance (ohms)
 T = Absolute temperature in Kelvin (K)
 B = System bandwidth (Hz)

Clearly, the available power associated with this mean square voltage is given by:

$$P = \frac{\overline{v^2}}{4R} = kTB \quad \text{Watts} \quad \text{Equation 2}$$

The bandwidth, B , is a function of the system, not the noise source. Therefore, it can be useful to define a parameter which depends only on the noise source and not on the system. This is known as the available power spectral density:

$$S = \frac{P}{B} = kT \quad \text{Watts} \quad \text{Equation 3}$$

In actual fact these expressions are only approximate since full quantum mechanical analysis yields an equivalent expression for the power spectral density of:

$$S = k\phi \quad \text{Equation 4}$$

$$\phi = T.P(f) \quad \text{Equation 5}$$

$$P(f) = \left(\frac{hf}{kT} \right) \left(e^{\frac{hf}{kT}} - 1 \right)^{-1} \quad \text{Equation 6}$$

where:

h is the Planck constant (6.626×10^{-34} Joule-secs)

f is the frequency (Hz)

ϕ has been referred to as the "quantum noise temperature"[3].

Unless the temperature, T , is very low or the frequency, f , is very high, the factor $P(f)$ is close to unity and Watts **Equation 3** and **Equation 4** become identical.

2.2 Shot Noise.

In an active device, the most important source of noise is shot noise [4] which arises from the fact that the charge carriers are discrete and are emitted randomly. This is most easily visualised in the context of a thermionic valve but applies equally to solid-state devices. Due to this, the instantaneous current varies about the mean current in a random manner which superimposes a noise like signal on the output of the device.

2.3 Flicker noise

Another cause of noise is flicker noise, also known as $1/f$ noise because the amplitude varies approximately inversely with frequency. As a consequence, it is rarely important at frequencies above a few kHz and it will not be considered further.

3 Definitions

The fundamental quantity measured when measuring noise is usually either a mean noise power or a mean square noise voltage. However, the relationship given in **Watts Equation 2** allows us to express the noise as an equivalent noise temperature and it is very often convenient to do this. If the equivalent thermal noise power from a source is $kT_e B$ then T_e is the equivalent available noise temperature of the source. Noise sources are very often specified as having a given value of the Excess Noise Ratio or ENR, usually expressed in decibels. The ENR is defined as:

$$ENR = 10 \log_{10} \left(\frac{T_e - T_0}{T_0} \right) \text{ dB} \quad \text{Equation 7}$$

where T_0 is the "standard" temperature of 290 K (17°C)

The definition given above is for available noise power into a conjugately matched load. In the past, some laboratories have measured noise power into a perfectly matched load giving an effective noise temperature T_e' and an effective value of ENR.

$$T_e' = T_e (1 - |\Gamma|^2) \quad \text{Equation 8}$$

$$ENR' = 10 \log_{10} \left(\frac{T_e' - T_0}{T_0} \right) \quad \text{Equation 9}$$

where Γ is the reflection coefficient of the noise source. The difference between ENR and ENR' is shown in

Figure 1.

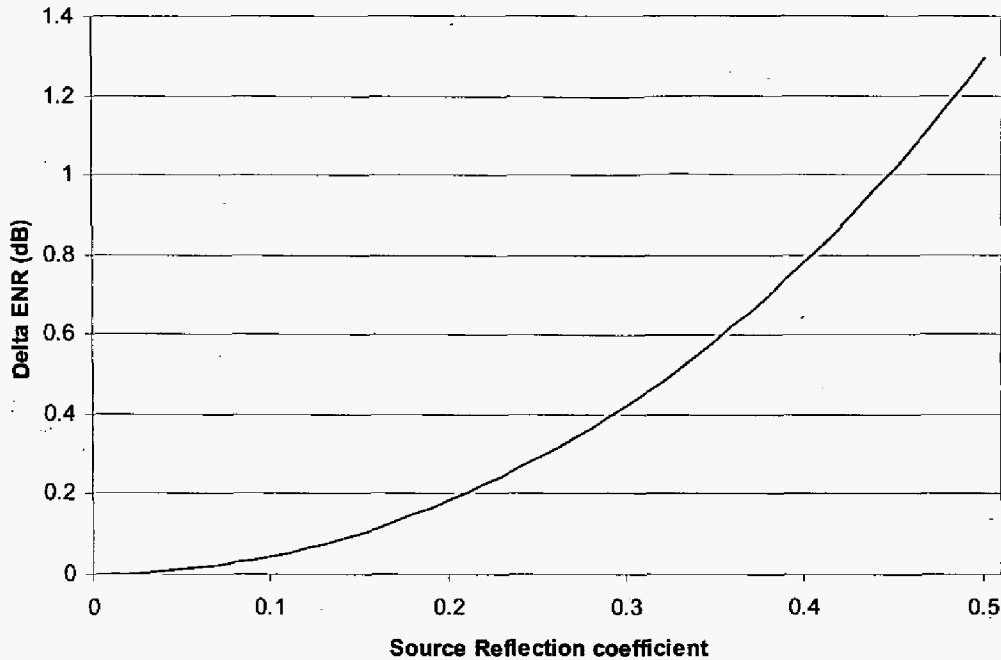


Figure 1

As can be seen, the error is small for reflection coefficients with small magnitude but becomes quite large as the reflection coefficient increases. Noise sources with very high values of ENR are often quite poorly matched so this could become important.

The noise performance of a receiver is usually specified as a noise factor or noise figure. The definition of noise figure can be found in several places, for example, reference 5, the Alliance for Telecommunications Industry Solutions. Excerpts from that definition state:

- It is determined by (a) measuring (determining) the ratio, usually expressed in dB, of the thermal noise power at the output, to that at the input, and (b) subtracting from that result, the gain, in dB, of the system.
- In some systems, e.g., heterodyne systems, total output noise power includes noise from other than thermal sources, such as spurious contributions from image-frequency transformation, but noise from these sources is not considered in determining the noise figure. In this example, the noise figure is determined only with respect to that noise that appears in the output via the principal frequency transformation of the system, and excludes noise that appears via the image frequency transformation.

In rare cases (most obviously radio astronomy) the principal frequency transformation may include both sidebands but usually only one sideband is considered. The terms "noise figure" and "noise factor" are normally considered to be synonymous although sometimes the term noise factor is used for the linear value (not in dB) while noise figure is used when the value is expressed logarithmically in dB. Adopting this distinction here we have:

$$F = \frac{N_o}{GkT_0 B} \quad \text{Equation 10}$$

where F is the noise factor, G is the gain and B is the bandwidth. Therefore, N_o , the total noise power from the output is:

$$N_o = GFkT_0 B \quad \text{Equation 11}$$

The input termination contributes an amount $GkT_0 B$, and so N_r , the noise contribution from the receiver itself is:

$$N_r = (F - 1)GkT_0 B \quad \text{Equation 12}$$

In situations where there is very little noise (eg radio astronomy or satellite ground stations) it is more common to use the equivalent input noise temperature. A definition is given at reference 6 and is as follows:

At a pair of terminals, the temperature of a passive system having an available noise power per unit bandwidth at a specified frequency equal to that of the actual terminals of a network

In most situations the pair of terminals chosen is at the input of the device so that all the noise at the output is referred back to the input and it is then imagined that all the noise is produced by a passive termination at temperature T_r . T_r is then the equivalent input noise temperature of the receiver.

The noise temperature and the noise factor can then be simply related. From Equation 12 we have:

$$N_i = (F - 1)kT_0 B$$

Equation 13

and from the definition of equivalent noise temperature we have:

$$N_i = kT_r B$$

Equation 14

and so:

$$T_r = (F - 1)T_0$$

Equation 15

The total noise temperature at the input is often referred to as the operating noise temperature which is given by:

$$T_{op} = T_s + T_r$$

Equation 16

Where T_s is the source temperature. Here we are assuming that there is no correlation between the source noise temperature and the receiver noise temperature and so the combined noise temperature is obtained by summing the noise temperatures.

4 Types of noise source

There are several types of noise source of which four are common. Two of these are particularly useful as primary standards of noise while the other two are more practical for general use.

4.1 Thermal noise sources

Thermal noise sources are very important because they are the type of noise source used throughout the world as primary standards of noise. To produce such a noise source, a microwave load is kept at a known temperature. In a perfect standard the transmission line between the non-ambient temperature and the ambient output would either have an infinitely sharp step change of temperature or it would have zero loss. If either of these idealisations occurred, calculation of the output

noise temperature of the device would be trivial. However, in practice, the transmission line cannot fulfil either of these requirements. Along the length of the transmission line through the transition from non-ambient to ambient, each infinitesimal section of the lossy line will both produce noise power proportional to its local temperature and absorb power incident upon it. To calculate the effect of this, measurements of the loss of the line must be made and an integration along the length of the line performed.

In the UK the majority of the primary standards used are hot standards operating at approximately 673 K and some are cold standards operating at 77 K. These are described in references 7, 8, 9 and 10, and are used at NPL to calibrate noise sources for customers worldwide. Commercial thermal noise sources have been available but the accuracy offered by these devices is limited by the attenuation measurements of the transition section and other factors and is not as good as what can be achieved from the devices at the National Standards Laboratories.

4.2 The temperature limited diode

This is not a particularly common noise source. It is formed by using a thermionic diode in the temperature-limited regime, where all the electrons emitted from the cathode reach the anode. In this situation the current has noise which is determined by shot noise statistics and is calculable – in other words, it can be used as a primary standard. However, due to effects such as transit time and inter-electrode capacitances, these devices have previously only been used for relatively low frequencies up to perhaps 300 MHz. The device is described in reference 11.

4.3 Gas discharge tubes

A gas discharge tube is an excellent broadband noise source. Tubes of this sort are described in reference 12. The noise signal is produced by the random acceleration and deceleration of electrons in the discharge as they collide with atoms, ions or molecules in the gas. In general, the gas used is argon, neon or xenon at low pressure. The noise temperature is typically around 10,000 K. Commercially available waveguide devices can still be obtained and, for the higher frequencies, these are very good sources. The tube containing the gas is mounted at an angle across the waveguide and the waveguide is usually terminated with a good load at one end while the other is the mounting flange for the device. The match of the device is usually excellent and the variation in the reflection coefficient between the "on" and the "off" state is very small. In a practical measurement the "on" state provides a high noise temperature while the "off" state provides a noise temperature which is at the physical temperature of the device ie close to ambient. These devices are obtainable up to

frequencies of 220 GHz [13]. These devices are not calculable and therefore require calibration before use. Once calibrated, they are relatively stable and will maintain their calibration for a considerable period if handled carefully.

4.4 Avalanche diode noise sources

A p-n junction which is reverse biased can produce noise through similar mechanisms to those in a gas discharge tube – ie the random acceleration and deceleration of the charges in the material [14]. Provided the current is sufficiently high, the noise produced is almost independent of frequency as shown in Figure 2

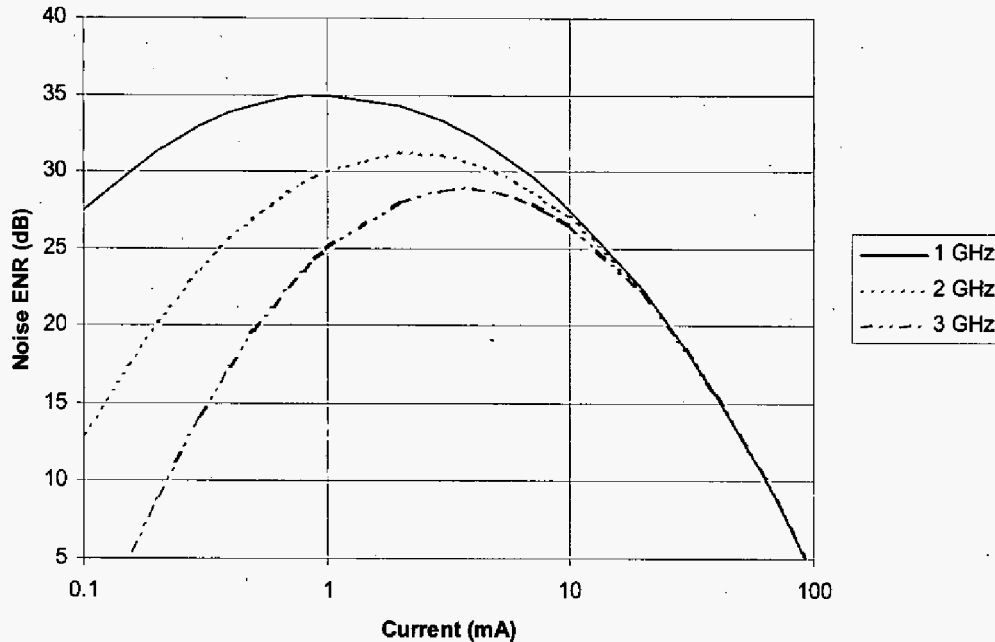


Figure 2

These devices are wideband, convenient, easy to use and are the preferred type of noise source for most applications. They are easily switched on and off rapidly, making measurements convenient. Recent developments include noise sources which contain a calibration table internally (on an EEPROM) and a temperature measuring device in the package so that the cold (or "off") temperature can be measured in situ rather than assumed to be ambient. As for the discharge tube, the "on" state has a high noise temperature while the "off" state has a noise temperature which is close to ambient. In general, the diode will produce a high ENR with a match which is poor and which has a considerable variation between the "on" state and the "off" state. To produce a lower value of ENR an attenuating pad is inserted which reduces the ENR and also improves the match and reduces its variation. These devices are not calculable and must be calibrated before use. They are reasonably stable although variations can occur. The condition of the connector is an important factor to consider when assessing the stability and repeatability of these devices.

5 Measuring noise

Instruments used to measure noise are classified under the general description of radiometers of which there are many types. Nowadays, a variety of instrument types can be configured to perform the measurement (eg noise figure analysers, spectrum analysers) but the actual operation is that of a radiometer and an understanding of the principle will allow the user to understand the way in which the measurement is made and the resulting limitations. There are many types of radiometer – the two most common are the total power radiometer and the Dicke (or switching) radiometer. In this document we will consider only the total power radiometer but those interested in the Dicke radiometer are referred to reference 15.

5.1 The Total Power Radiometer

The simple form of the total power radiometer is shown in Figure 3.

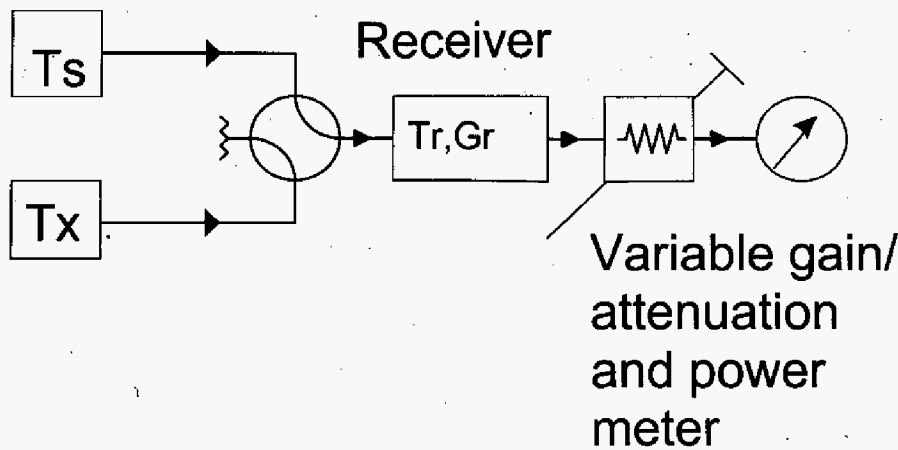


Figure 3

In the total power radiometer, the final output is a power reading which simply measures the total power coming from the input and from the noise generated in the receiver. We can assume that these are not correlated and that the total power is given by a simple sum of the individual powers.

When the first device, T_s , (generally a standard), is attached to the input we have:

$$(T_s + T_r)G_r = P_1 \quad \text{Equation 17}$$

and when the second device T_x , (either an ambient standard or the unknown), is attached we have:

$$(T_x + T_r)G_r = P_2 \quad \text{Equation 18}$$

The detection system is likely to have non-linearities and so it is good practice to include a calibrated attenuator in the system to ensure that the output power is held at a constant level and so:

$$(T_s + T_r)G_r A_1 = (T_x + T_r)G_r A_2 \quad \text{Equation 19}$$

Commonly, the ratio of the attenuator settings, A_1/A_2 is called the Y-factor, Y and so, re-arranging, we have:

$$\frac{(T_x + T_r)}{(T_s + T_r)} = Y \quad \text{Equation 20}$$

Equation 20 can be re-arranged to give either T_x or T_r depending on whether one is calibrating the receiver or the unknown.

$$T_x = YT_s + T_r(Y - 1) \quad \text{Equation 21}$$

or

$$T_r = \frac{(T_x - YT_s)}{(Y - 1)} \quad \text{Equation 22}$$

The receiver noise temperature must be obtained first through the use of two known noise temperatures, generally a 'hot' standard and, for convenience, an ambient load. Once this is done, the unknown noise temperature may be obtained using either the standard or an ambient device.

A usable total power radiometer requires very stable gain throughout the system since the gain must remain constant throughout the measurement. Very high values of gain (perhaps up to 100 dB) will be required since we are dealing with very small input powers; for example, a thermal noise source at 290 K has a power spectral density of -204 dBW/Hz. If the gain is not stable then measurement errors will result. In the past, this was difficult to achieve and was one of the motivations for the development of the Dicke radiometer which relies on a stable reference device. However, more recently, adequate stability has been achieved and more recent radiometers tend to be total power because the total power radiometer is more sensitive. Radiometer sensitivity is the topic of the next section.

5.2 Radiometer sensitivity

The sensitivity of the radiometer is limited by random fluctuations in the final output [16, 17]. These fluctuations have an RMS value referred to the input given by:

$$\Delta T_{\min} = \frac{aT_{op}}{\sqrt{B\tau}} \quad \text{Equation 23}$$

Where:

B is the pre-detector bandwidth

τ is the post-detector time constant

T_{op} is as defined in Equation 16

a is a constant which depends on the radiometer design

ΔT_{min} is the minimum resolvable temperature difference

The constant a will be unity for a total power radiometer and between 2 and 3 for a Dicke radiometer depending on the type of modulation and detection used. It is for this reason that a total power radiometer is to be preferred if adequate gain stability can be achieved.

6 Measurement accuracy

We have seen earlier that a total power radiometer can be used to measure an unknown noise temperature provided that two different standards of noise temperature are available (Equation 21 and Equation 22). If we now assume that one of these sources is "hot" (ie a calibrated noise source) and the other is "cold" (ie an ambient temperature load) and denote these by T_h and T_c respectively, we can re-write Equation 22 as:

$$T_r = \frac{(T_h - YT_c)}{(Y - 1)}$$

Equation 24

Wherever possible, the noise temperature being measured should be somewhere between the two standards. Rough guidelines for the choice of noise standards are given by:

$$T_r = \sqrt{T_h T_c} ; \text{ or } 4 \leq \frac{T_h}{T_c} \leq 10$$

Equation 25

Commercial solid-state noise sources are typically either 5 dB ENR (about 1000 K) or 15 dB ENR (about 10,000 K) in the "on" state. In the "off" state, they have a temperature close to the ambient temperature and so the measurements can be made by connecting only one noise source to the device under test. For very low noise devices, a cold load

might provide better measurement uncertainties. In order to see this, it is best to derive the uncertainties with respect to each input variable. This is done by partial differentiation:

$$\Delta T_{r1} = \frac{1}{Y - 1} \Delta T_h$$

$$\Delta T_{r2} = \frac{-1}{Y - 1} \Delta T_c$$

$$\Delta T_{r3} = \frac{(T_c - T_h)}{(Y - 1)^2} \Delta Y$$

These are the type "B" uncertainties in the terminology defined in the appropriate guide [18] and the type "A" uncertainties must be added to give an overall uncertainty. The type "A" uncertainties are the uncertainties determined by statistical means. In the case of a noise radiometer, we can obtain an approximation to the magnitude of the type "A" uncertainties from Equation 23. This gives:

$$\Delta T_{r4} = \left[\frac{(T_h + T_r)^2}{B\tau} + \frac{(T_c + T_r)^2}{B\tau} \right]^{\frac{1}{2}}$$

and the total uncertainty is then:

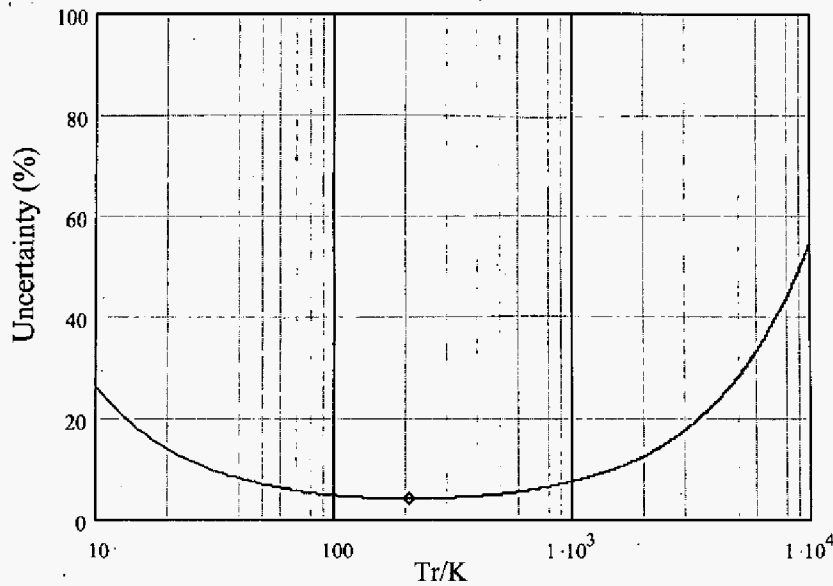
$$\Delta T_{tot} = \left[\Delta T_{r1}^2 + \Delta T_{r2}^2 + \Delta T_{r3}^2 + \Delta T_{r4}^2 \right]^{\frac{1}{2}}$$

This relation has been used to derive the data shown in Figure 4 and Figure 5. In these figures, T_a denotes an ambient load used as one of the temperature references and T_v is a variable temperature noise source used as the other reference. ΔT_v , the uncertainty in T_v is assumed to be 2% of T_v and ΔT_a , the uncertainty in T_a , is fixed at 0.5 K. The other parameters are:

$$\Delta Y = 0.05 \text{ dB}$$

$$B = 2 \text{ MHz}$$

$$\tau = 1 \text{ second}$$



$$T_v = 77$$

$$\Delta T_v = 1.54$$

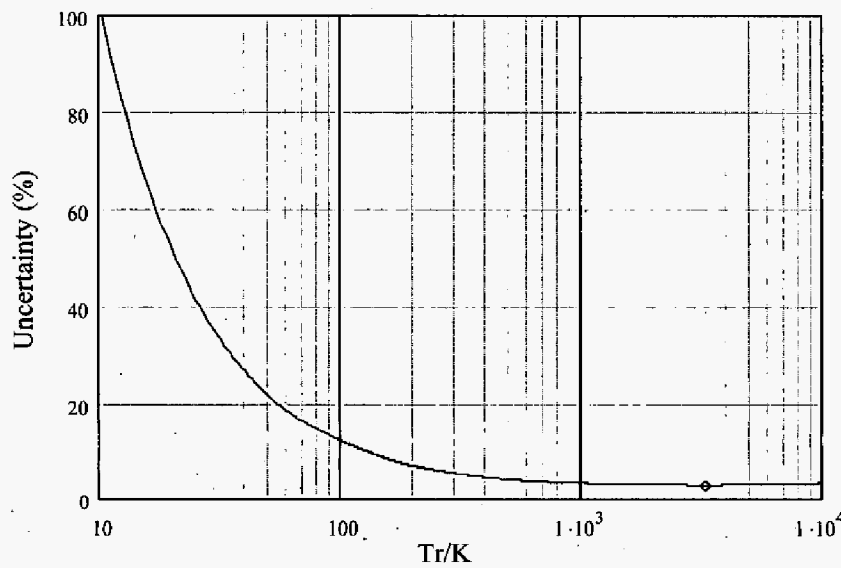
$$T_a = 296$$

$$\Delta T_a = 0.5$$

$$T_{rmin} = 205$$

$$MinError = 4 \%$$

Figure 4



$$T_v = 1 \times 10^4$$

$$\Delta T_v = 200$$

$$T_a = 296$$

$$\Delta T_a = 0.5$$

$$T_{rmin} = 3.259 \times 10^3$$

$$MinError = 3 \%$$

Figure 5

It is clear that the choice of suitable noise temperature references is very important if low uncertainties are desired, particularly for the measurement of very low noise temperatures.

When measuring a mixer or other device with frequency conversion, it is important to consider the image frequency. Noise sources are broadband devices so, in the absence of any image rejection, the noise source will provide an input signal to both sidebands. The definition of noise figure requires that only the principal frequency transformation is measured. If both sidebands are measured due to the lack of any image rejection, the measurement

will be in error. When the loss for both sidebands is equal this error will be a factor of 2, or 3 dB and so we have [19, 20]:

$$F_{ssb} = 2F_{dsb} \quad \text{whence} \quad T_{ssb} = 2T_{dsb} + 290$$

Equation 26

but in many situations, this will not be the case and so the error will be large and difficult to quantify.

6.1 Cascaded receivers

If we have a cascade of receivers or amplifiers as shown in below:

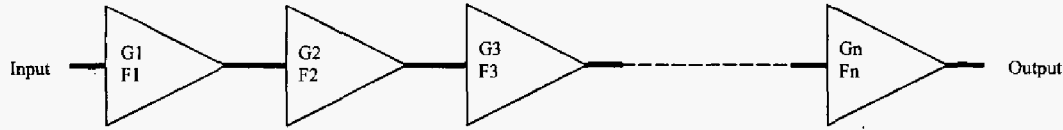


Figure 6

Then the overall noise figure can be calculated using the expression:

$$F_t = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \dots + \frac{(F_n - 1)}{(G_1 G_2 \dots G_{n-1})}$$

Equation 27

or, in terms of noise temperature:

$$T_t = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots + \frac{T_n}{(G_1 G_2 \dots G_{n-1})}$$

Equation 28

If G_1 is large then the higher order terms can be ignored. It is also evident that the quality of this first amplifier has a large bearing on the performance of the overall system.

When we want to design a cascade with the lowest possible noise figure a parameter known as the noise measure is defined [21] as:

$$M = \frac{(F - 1)}{\left(1 - \frac{1}{G}\right)}$$

Equation 29

The amplifier with the lowest value of noise measure should come first in the cascade.

6.2 Noise from passive two-ports

Any real system will contain passive devices which will have some loss. These devices will be noise sources with a noise temperature equivalent to their physical temperature. Consider the arrangement in Figure 7 below:

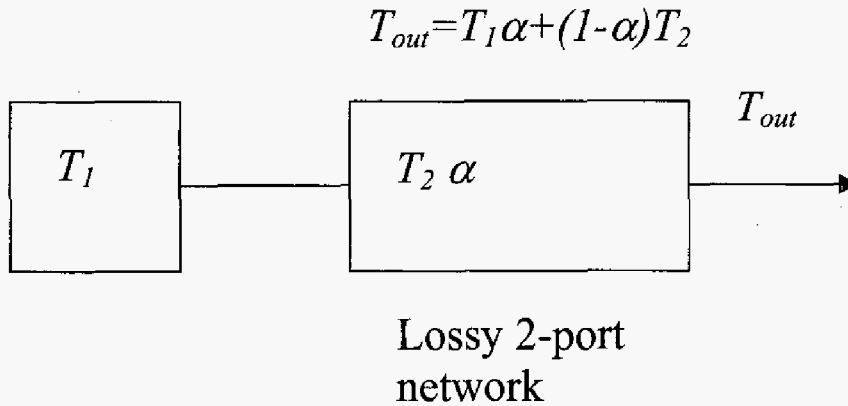


Figure 7

Here the transmission coefficient of the two-port is denoted by α and hence its loss is $(1 - \alpha)$. It will therefore have a noise temperature of $(1 - \alpha)T_2$. The incident noise from the source on the input will be attenuated by the two-port to a value of $T_1\alpha$ and so

the total noise temperature (assuming no correlation) is:

$$T_{out} = T_1\alpha + (1 - \alpha)T_2$$

Equation 30

If this is applied to the case in Figure 8 below:

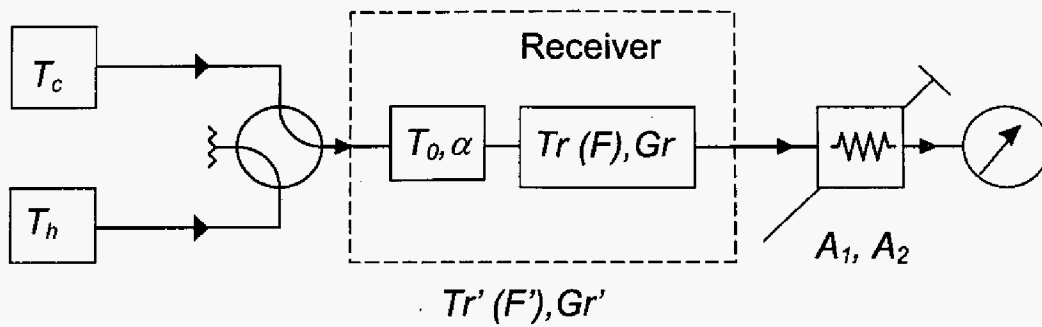


Figure 8

where a radiometer is preceded by a lossy two-port then we can use Equation 19 and Equation 30 to write:

$$[T_c \alpha + (1 - \alpha)T_0 + T_r]G_r A_1 = [T_h \alpha + (1 - \alpha)T_0 + T_r]G_r A_2$$

Now, letting $Y = A_1/A_2$ and re-arranging:

$$T_r = \left[\alpha \frac{(T_h - Y T_c)}{(Y - 1)} \right] - (1 - \alpha)T_0$$

Equation 31

If we now treat the lossy network and the radiometer as a single unit (ie outside the dotted box in Figure 8):

$$T_r' = \frac{(T_h - Y T_c)}{(Y - 1)}$$

Equation 32

Using $F = \left(\frac{T_r}{T_0} \right) + 1$ and Equation 31 we obtain:

$$F = \alpha \left[\frac{(T_h - Y T_c)}{(Y - 1)T_0} + 1 \right]$$

Equation 33

From Equation 32 we have

$$F' = \frac{(T_h - Y T_c)}{(Y - 1)T_0} + 1$$

Equation 34

and so $F = \alpha F'$ but $\alpha < 1$ and so $F' \text{ dB} = F \text{ dB} + \alpha \text{ dB}$

Equation 35

When measured in dB, any losses in front of the radiometer add directly to the noise temperature of the radiometer. This is only strictly true if the lossy two-port is at the standard temperature of 290 K. If

it is at any other temperature a new expression for F in Equation 33 must be derived using the same procedure.

Mismatch Effects

Up to now, the effect of mismatch has been ignored in the analyses. In reality, there will usually be mismatches and these must be considered.

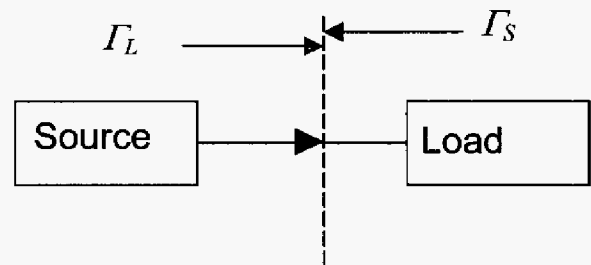


Figure 9

$$\text{Mismatch Factor} = \frac{\text{Power Delivered to Load}}{\text{Power Available from Source}}$$

$$M = \frac{(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_S \Gamma_L|^2}$$

Noise is affected in two ways by mismatches. Firstly, in common with all other microwave signals, there will be mismatch loss [22]. Secondly, and more subtly, the noise temperature of a receiver is affected by the input impedance. This is because noise emanating from the first active device in the direction of the input will be correlated with the noise emanating from it in the direction of the output. When this noise is reflected off the input mismatch it will still be partially correlated with the noise going towards the output and so the two noise powers cannot be simply summed, a more complex analysis is required. However, simple steps can be taken to reduce, or eliminate this effect. In the past, tuners were frequently used but this is less common since manual tuners are slow and therefore

expensive in operator time and automatic tuners are expensive in capital cost. Therefore the use of

isolators is more common now.

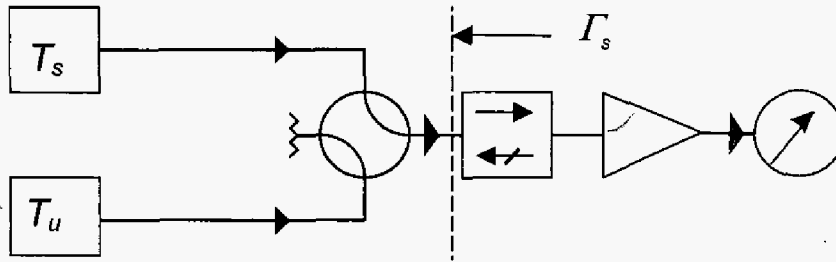


Figure 10

In this situation the radiometer sees a constant input impedance for both switch positions and so is not affected by any variation in the reflection coefficients of the two noise sources. There is still mismatch loss at the input port and so accurate noise measurements require the measurement of the complex reflection coefficients of both the isolator input and of the noise sources. Some accurate noise systems incorporate instrumentation to allow this to be done in-situ [23].

7.1 Measurement of receivers and amplifiers

For many people, measurement of noise means measurement of the noise figure of an amplifier. It is important to realize that this figure is not a unique parameter – the noise figure is an insertion measurement which depends on the source impedance the amplifier sees when it is measured. A measurement made with a different source impedance will yield a different result. Fortunately, the effect is often quite small, but it should not be overlooked. Two options exist:

- Measure the amplifier in a defined impedance environment and inform the user what the measurement conditions are.
- Provide the full complex noise parameters so that the user can calculate the noise for any impedance configuration.

In the past, the first option was often adopted and the amplifier was measured in a perfectly matched environment. However, increasingly, users wish to obtain the very best performance from their amplifiers and, to do this, full knowledge of the complex amplifier noise parameters is required.

There have been several different representations of the complex amplifier noise parameters. These all

provide the same information and so one set can readily be converted to another. The most common (because they are the most useful to the practicing engineer) are the parameters defined by Rothe and Dalke [24]. The most familiar form is:

$$F = F_{\min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2$$

Equation 36

in terms of noise factor and admittances. It can be written in terms of noise temperatures and reflection coefficients as:

$$T_r = T_{\min} + \frac{4T_0 R_n}{Z_0} \left[\frac{|\Gamma_s - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_s|^2)} \right]$$

Equation 37

In Equation 37 the noise temperature, T_r , will reach its minimum value, T_{\min} , when the reflection coefficient at the source, Γ_s , is at its optimum value, Γ_{opt} . Similarly, in Equation 36 the noise factor, F , will reach its minimum value F_{\min} when the source admittance, Y_s , is at its optimum value, Y_{opt} . In both equations R_n is the noise resistance and determines how rapidly the noise increase as the source admittance or reflection moves away from optimum. In Equation 37 T_0 is the usual standard noise temperature 290 K, Z_0 is the characteristic impedance of the transmission line and in Equation 36 G_s is the conductance of the transmission line.

The issue of correlated noise has been touched upon several times already. The following description [25] may make this clearer.

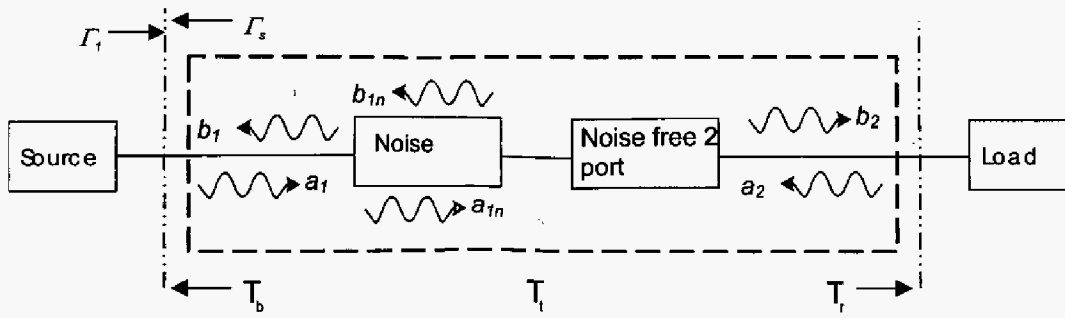


Figure 11

Referring to Figure 11 above, the amplifier (enclosed within the dotted box) can conceptually be split into a perfect, noise free two-port with a two-port noise source. The latter may be on the input or the output; here it is assumed to be on the input. Noise waves, a_{1n} and b_{1n} are produced and propagate in each direction. Since these are produced in the same place, in the same way, they are correlated. After b_{1n} is reflected from the source, there will still be some degree of correlation with a_{1n} and so the output noise temperature, T_l cannot be correctly evaluated without accounting for this. The amount of correlation depends on both the magnitude and the phase of the source reflection coefficient. It should be noted that the apparent noise temperature at the input, T_b , is different and can be well below ambient [26, 27, 28].

The total noise incident on the two-port (remembering the noise generated within the two-port is referred to the input) will be:

$$N_t = N_s + N(a_{1n}) + \Gamma_s N(b_{1n})$$

where:

N_t is the noise at the output of the receiver

$N(a_{1n})$ is the noise due to the forward going noise wave

$N(b_{1n})$ is the noise due to the reverse going noise wave

Γ_s is the source reflection coefficient

If we assume no correlation between the noise source on the input and the noise generated by the receiver then:

$$\overline{|N_t|^2} = \overline{|N_s|^2} + \overline{|N(a_{1n})|^2} + |\Gamma_s|^2 \overline{|N(b_{1n})|^2} + 2 \operatorname{Re}(\Gamma_s N(a_{1n})^* N(b_{1n}))$$

where, for example, $\overline{|N_t|^2}$ denotes the mean square value of N_t and the asterisk denotes the complex conjugate.

Recalling the earlier definitions we can say:

$$\overline{|N_t|^2} = kT_l B$$

$$\overline{|N_s|^2} = kT_s B$$

$$\overline{|N(a_{1n})|^2} = kT_r B$$

$$\overline{|N(b_{1n})|^2} = kT_b B$$

Now define a new parameter Γ' as $\Gamma' = \frac{(\Gamma_s - \Gamma_l^*)}{(1 - \Gamma_s \Gamma_l)}$

Equation 38

which has the property $M_s = 1 - |\Gamma_s|^2$ where M_s is the mismatch factor at the input of the amplifier.

Dividing throughout by kB and introducing a complex parameter T_c which represents the degree of correlation between T_r and T_b gives:

$$T_l = T_s + T_r + |\Gamma'|^2 T_b + 2 \operatorname{Re}(T_c \Gamma')$$

Equation 39

The terms T_r , T_b and T_c are the parameters defined by Meys [29].

Equation 40

Measurement of the noise parameters of an amplifier expressed in any of the various forms can be done relatively easily by measuring the total noise power output with a variety of input terminations at least one of which must be at a different noise temperature to the others. Usually

this is done by using a noise source which provides a hot noise temperature (on) and ambient noise temperature (off) at a reflection close to a match and a set of mismatches which provide ambient terminations away from a match. Choice of the values for the mismatches is not trivial if a low uncertainty measurement is to be achieved with the minimum of mismatches [30, 31]. Obviously, whatever input sources are used, their reflection coefficient must be measured and used to calculate the noise parameters and this, alone, is enough to make the whole measurement much more time consuming, complex and expensive in terms of equipment.

8 Automated noise measurements

The great majority of noise measurements are made using some sort of automated system, most commonly a noise figure analyzer. The first widely available instruments of this kind were introduced more than two decades ago and, although there have been many improvements in usability and in accuracy since then, the basic principles have not changed and are, indeed, those of the total power radiometer already described.

8.1 Noise figure meters or analyzers

The basic block diagram of a noise figure analyzer is shown below in Figure 12.

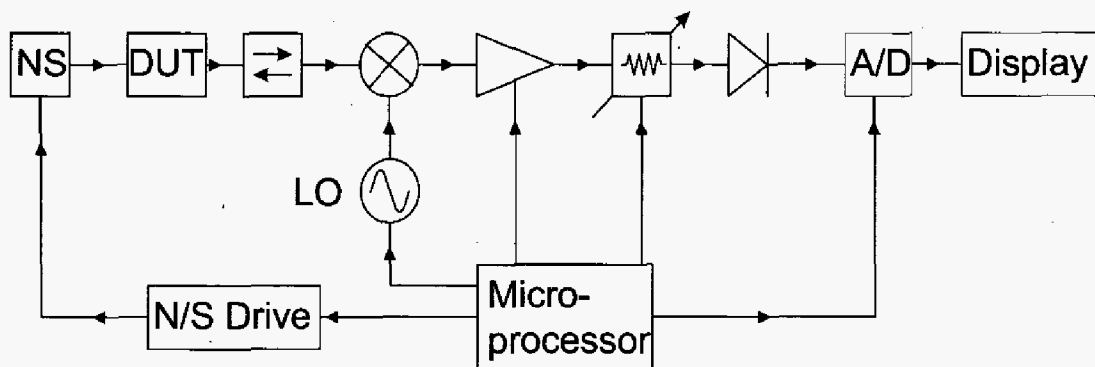


Figure 12

Modern instruments will cover a wide bandwidth (eg 10 MHz to 26.5 GHz) in a single unit and will have inbuilt filtering to avoid image problems. The most modern instruments have selectable measurement bandwidth (achieved by digital signal processing). The calculation of the receiver noise temperature is performed by the processor and then used to correct the DUT measurements. Often a variety of parameters may be displayed, e.g. gain and noise figure, but it is important to remember that the only measurement which is actually made by the instrument is the Y-factor. The sources of uncertainty in the measurement are the same as those made by other methods described earlier.

The block diagram above shows an isolator on the input. This component is not, in fact, generally part of the instrument but should be added externally for best uncertainty for reasons described earlier. The mixer shown is an internal component, but external mixers may be added to extend the frequency range. If this is done then the user may have to be concerned with image rejection.

8.2 On-wafer measurements

Increasingly, noise measurements are being performed on-wafer. There are many difficulties with this, in common with all on-wafer measurements. The main interest is in the

measurement of the full complex noise parameters and so variable reflection coefficients are required. These are usually achieved using tuners, either solid state or mechanical. The majority of these systems use off-wafer noise sources, off-wafer automatic tuners and off-wafer instrumentation (noise figure analyzer and network analyzer). A probing station is used to link all these to the on-wafer devices [32]. This is a complex and error prone method of measurement. Discussion of the intricacies of on-wafer measurement is not the scope of this document and so will not be considered further here.

Comparative measurements on-wafer are much easier to perform and these are fairly routinely performed. The measurements can be checked by including passive devices such as an attenuator on the wafer [33]. Other workers have proposed a passive device based upon a Lange coupler which also has calculable noise characteristics and in addition is so designed that its scattering coefficients are similar to the FET structures often being investigated [34].

9 Conclusion

This document has attempted to give an overview of noise metrology from primary standards to practical systems. The view is, of necessity, partial and brief. There are other works on measurements which also

include a discussion of noise metrology. The interested reader is referred in particular to [35] and [36].

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