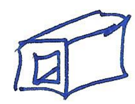
Microwave Basics

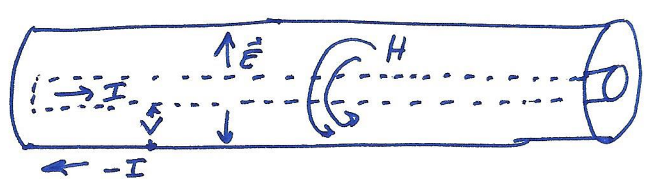
Microwave energy is transported by means of transmission lines.

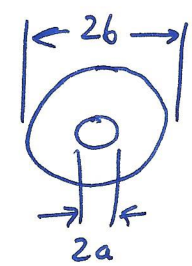
TEM — No longitudinal component of $\vec{E}$ or $\vec{B}$
Requires two conductors

TE, TM — Present in waveguide (single conductor)



Coaxial Cables are the most common form of transmission line





The radial electric field is given by

$$E = \frac{V}{\ln(b/a)} \frac{1}{r} \quad (V/m)$$

The axial magnetic field is given by

$$H = \frac{I}{2\pi} \frac{1}{r} \qquad (A/m)$$

Fig. 1.1-3(a) is a nice picture of the coaxial TEM mode.

2

The speed of light in the coaxial cable is

$$V_{ph} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0}}$$

ε_r = relative permittivity
μ_r = relative permeability

$$\lambda f = V_{ph} \qquad \lambda_{TEM} = \frac{\lambda_{vac}}{\sqrt{\varepsilon_r}}$$

$$V(t) = |V| e^{i(2\pi f t + \phi)}$$
$$V(x,t) = |V| e^{i(2\pi f t + \phi - \beta x)}$$

$$\beta = \frac{2\pi}{\lambda}$$

If a wave propogates a distance L, its phase advances by $\beta L = 2\pi L/\lambda$.

1.3 Characteristic Impedance

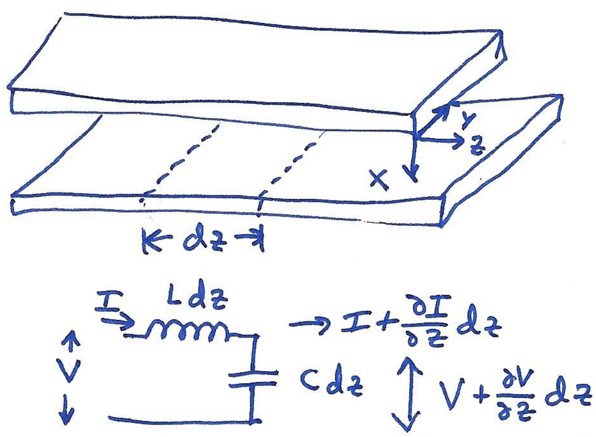
A property of a transmission line is its characteristic impedance. It is defined as the ratio of voltage to current at a given point;

$$Z_c = \left.\frac{V}{I}\right|_{point} \quad (\Omega)$$

True for a travelling wave. The characteristic impedance for a standing wave is not constant, since that is a combination of two waves travelling in opposite directions. Their

$$Z_c(x) = \frac{V_{TOTAL}(x)}{I_{TOTAL}(x)}$$ varies down the line.

~~For a travelling wave;~~
Derive the equations for a travelling wave on a transmission line;



L = inductance / length
C = capacitance / length

Inductance is associated with the flux produced by the oppositely directed currents in the pair of conductors. When the currents vary in time, there is a voltage change along the line.

The change in voltage is the inductance times the time rate of change of the current.

$$\text{Voltage change} = \frac{\partial V}{\partial z} dz = -(L\,dz)\frac{\partial I}{\partial t}$$

A change in current along the line is merely the ~~current~~ voltage that is shunted across the distributed capacitance. The rate of decrease of the current with distance is the capacitance times the time rate of change of the voltage

$$\text{Current change} = \frac{\partial I}{\partial z}\,dz = -(C\,dz)\frac{\partial V}{\partial t}$$

So one has the two equations

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

By taking further partial derivatives, one can get a wave equation for either V or I;

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} = \frac{1}{V_{ph}^2}\frac{\partial^2 V}{\partial t^2}$$

$$V_{ph} = \frac{1}{\sqrt{LC}}$$

also $$\frac{\partial^2 I}{\partial z^2} = \frac{1}{V_{ph}^2}\frac{\partial^2 I}{\partial t^2}$$

There are solutions of the form

$$V(z,t) \sim Ae^{i(t - z/V_{ph})} + Be^{i(t + z/V_{ph})}$$

To find the current one uses one of the above equations;

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$$

$$\frac{i}{V_{ph}}\left(-Ae^{i(t-z/V_{ph})} + Be^{i(t+z/V_{ph})}\right) = -L\frac{\partial I}{\partial t}$$

$$I(z,t) = \frac{-1}{LV_{ph}}\left(-Ae^{i(t-z/V_{ph})} + Be^{i(t+z/V_{ph})}\right)$$

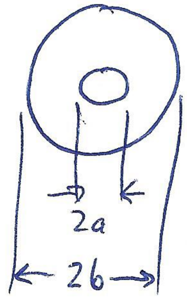
So

$$I \sim \frac{V}{LV_{ph}} \equiv \frac{V}{Z_0}$$

$$Z_0 = LV_{ph} = \sqrt{\frac{L}{C}}$$ The characteristic impedance.

For a Coaxial Line

$$C = \frac{q}{\Phi_{Diff}}$$



$$\Phi = -\int_a^b E_r\,dr = -\int_a^b \frac{q}{2\pi\varepsilon r} = \frac{-q}{2\pi\varepsilon}\ln(b/a)$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

$$L \equiv \frac{1}{I}\int_S \vec{B}\cdot d\vec{S}$$ But $$H_\phi = \frac{I}{2\pi r}$$

$$L = \frac{1}{I}\int_S \mu\frac{I}{2\pi r}\,dr\,dl = \frac{\mu}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu}{2\pi}\ln(b/a)$$

So for a coaxial cable;

$$V_{ph} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\mu}{2\pi} 2\pi\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$ Same as free space!

and

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{2\pi}\ln(b/a)}{2\pi\varepsilon/\ln(b/a)}} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\ln(b/a)}{2\pi}$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$
$$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$$

$$Z_0^{\text{Free Space}} = 376.8\,\Omega$$ Antenna must match this.

$$Z_0^{.085\ \text{Coax}} = 376.8\,\Omega\,\frac{1}{\sqrt{2}}\,\frac{\ln(.033/.010)}{2\pi}$$

$$= 50.6\,\Omega$$ Universal Standard

Losses and Attenuation:

Microwave signals vary in magnitude over many orders, so it is convenient to use logarithms to discuss their losses.

$$\text{Loss (nepers)} = \frac{1}{2}\ln\frac{P_1}{P_2}$$ $$\text{loss (dB)} = 8.686\ \text{loss (nepers)}$$

$$\text{Loss (dB)} = 10\log\frac{P_1}{P_2}$$

Suppose half of the power is lost in the transmission line

$$P_2 = P_1/2$$

$$\text{Loss (dB)} = 10 \log 2 = 3.01 \text{ dB}$$

3 dB is half power

Note that since $P \sim E^2$ or B^2

$$\text{Loss}_{E,B} \text{ (dB)} = 20 \log \frac{E_1}{E_2}$$

The attenuation describes the loss in power per unit length. α (dB/length)

$$\frac{P_2}{P_1} = 10^{-\frac{1}{10}\alpha \times \text{Length}}$$

$$\frac{V_2}{V_1} = \frac{I_2}{I_1} = 10^{-\frac{1}{20}\alpha \times \text{Length}}$$

For a normal metal transmission line $\alpha \sim \sqrt{f}$

For a superconducting transmission line $\alpha \sim f^2$

Dielectric Loss

$$Z_c = \frac{1}{i\omega C} \qquad \sigma \sim i\omega C$$

$$\vec{J} = \sigma \vec{E}$$

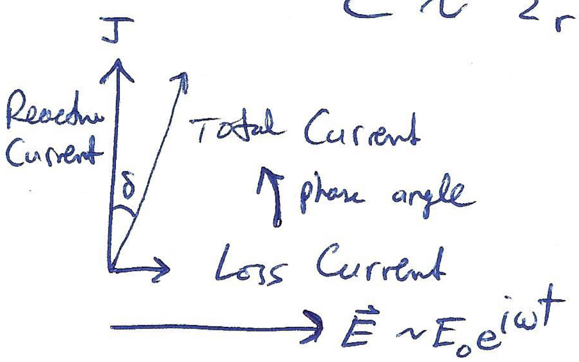
$$\vec{J} = i\omega C \vec{E}$$

So the displacement current leads the voltage by 90° in an ideal lossless capacitor. If the dielectric is lossy, there is a real part of the conductivity, and there is a component of current in phase with the voltage

$$\vec{J} = (i\omega C + \sigma_{real})\vec{E}$$

$$C \sim \varepsilon_r \rightarrow \varepsilon_r' - i\varepsilon_r''$$

Complex Dielectric Constant



$$\tan\delta = \frac{\text{Loss Current}}{\text{Reactive Current}}$$

$$= \varepsilon_r''/\varepsilon_r'$$

Attenuation in a dielectric is given by

$$\alpha_{diel} = \pi V_{ph} f \tan\delta \qquad (\text{nepers}/m)$$

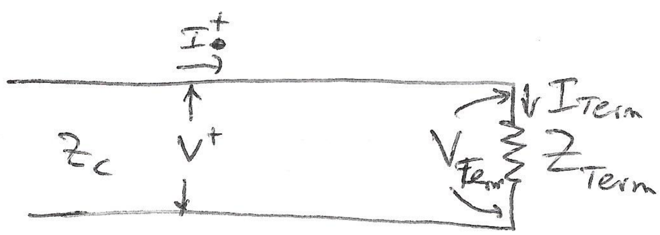
$$\sim f^1$$

Standing Waves

Before we were talking about travelling waves. But in reality, we have waves travelling in both directions on the transmission line → standing waves.

Any discontinuity in the uniform transmission line gives rise to standing waves

In microwave circuits the concepts of voltage and current are meaningful only when a single mode is propagating. Must establish a reference plane. The ratio of the voltage to current at that plane is defined as the device's impedance.



$$\frac{V^+}{I^+} = Z_c$$

For just one travelling wave going to the right.

$$\frac{V_{term}}{I_{Term}} = Z_{Term}$$

at the terminating impedance

The only way for $V^+ = V_{Term}$ and $I^+ = I_{Term}$ is if $Z_c = Z_{Term}$, that is the termination must have the same impedance as the coaxial cable.

If $Z_{Term} \neq Z_c$, then there must be a reflected wave. In that case

$$\frac{V^-}{I^-} = -Z_c$$

The total voltage and current are then

$$V = V^+ + V^-$$
$$I = I^+ + I^-$$

Require continuity at the terminating impedance

$$(V^+ + V^-)_{at\ Term} = V_{Term}$$
$$(I^+ + I^-)_{at\ Term} = I_{Term}$$

Combine these equations

$$V^+\big|_T + V^-\big|_T = Z_{Term}\, I_{Term}$$

$$\cancel{\frac{V^+}{I^+} - \frac{V^-}{I^-} = 2Z_c} \qquad \cancel{\frac{V^+}{I^+} + \frac{V^-}{I^-} = 0}$$

$$\cancel{V^+ - \frac{I^+}{I^-} V^- = 2Z_c I^+} \qquad \cancel{V^+ + \frac{I^+}{I^-} V^- = 0}$$

$$\frac{I^+}{V^+} = \frac{1}{Z_c} \qquad \frac{I^-}{V^-} = \frac{-1}{Z_c}$$

$$I^+ + I^- = \frac{1}{Z_c}(V^+ - V^-)$$

$$V^+ + V^- = Z_T/Z_c\,(V^+ - V^-)$$

$$V^+\left(1-\frac{Z_T}{Z_c}\right)+V^-\left(1+\frac{Z_T}{Z_c}\right)=0$$

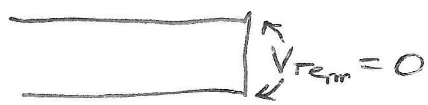
$$\boxed{V^- = V^+\left(\frac{-1+Z_T/Z_c}{1+Z_T/Z_c}\right)}$$

Ex Suppose $Z_{Term} = Z_c$, then

$$V^- = V^+\left(\frac{-1+1}{1+1}\right)=0$$

No reflection from a perfectly terminated line!

Ex Suppose there is a short circuit at the load; i.e. $Z_{Term}=0$



Then

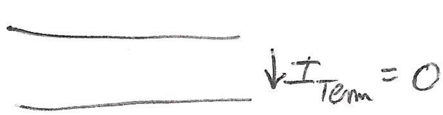
$$V^- = V^+\left(\frac{-1}{1}\right) = -V^+$$

and all of the signal is reflected, with a $180°$ phase shift!

Ex Suppose the transmission line ends with an "open" i.e. $Z_{Term}=\infty$, then

$$V^- = V^+(1)$$

This produces a complete reflection with no phase shift.



(Ex) Suppose the transmission line is left open to free space, $Z_{Term} = 377\,\Omega$

$$V^- = V^+ \left(\frac{377/50 - 1}{377/50 + 1} \right) = .76\, V^+$$

$$P^- = .586\, P^+ \qquad \Gamma = .76 \text{ Reflection coeff.}$$

<u>Reflection Coefficient</u>: Ratio of the reflected to incident voltages

$$\Gamma = \frac{V^-}{V^+}$$

so that

$$\Gamma = \frac{Z_T/Z_c - 1}{Z_T/Z_c + 1} \equiv |\Gamma| e^{i\theta}$$

θ is the angle by which the reflected voltage leads the incident voltage.

$$0 \le |\Gamma| \le 1$$

<u>Return Loss</u>

R compares the power in the reflected wave to the power in the forward wave

$$R(dB) = 10 \log_{10} \frac{\text{Incident Power}}{\text{Reflected Power}} = 10 \log_{10} \frac{|V^+|^2}{|V^-|^2} = 20 \log_{10} \frac{1}{|\Gamma|}$$

Totally reflecting termination $\rightarrow R = 0$

Totally absorbing termination $\rightarrow R \rightarrow \infty$.

Transforming voltages on a transmission line

Suppose we know the voltage and current at w_1, and we wish to calculate it at W_2.

The forward voltage changes in magnitude $e^{\alpha(W_2-W_1)}$

The forward voltage changes in phase by $\beta(W_2-W_1)$

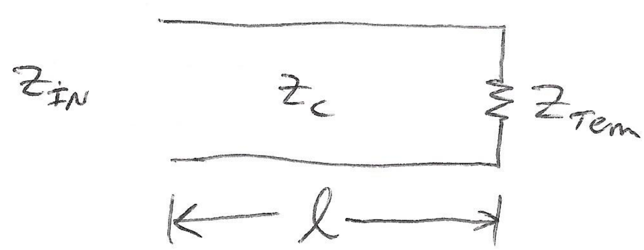
$$V^+(w_2) = V^+(W_1)\, e^{\alpha(W_2-W_1)}\, e^{i\beta(W_2-W_1)}$$

$$V^-(w_2) = V^-(W_1)\, e^{-\alpha(W_2-W_1)}\, e^{-i\beta(W_2-W_1)}$$

The reflection coefficient goes as

$$\Gamma(w_2) = \Gamma(W_1) e^{-2\alpha(W_2-W_1)}\, e^{-2\beta(W_2-W_1)}$$

Transformer Rules



$$\bar{Z}_{IN} = \frac{\bar{Z}_{Term} + i\tan\beta l}{1 + i\,\bar{Z}_{Term}\tan\beta l} \qquad \bar{Z} = Z/Z_c$$

The transmission line of length l transforms the impedance of the termination to Z_{IN}.

Examine the properties of various transmission lines

Ex Let $Z_{Term} = Z_c$, then $\bar{Z}_{IN} = 1$ or $Z_{IN} = Z_c$ for any length of line.
A perfectly terminated line has the impedance of the transmission line

Ex Suppose $l = \lambda/2$ or $n\lambda/2$, then $\beta l = \pi$, $\tan \beta l = 0$ and

$$\bar{Z}_{IN} = \bar{Z}_{Term}$$

so the transmission line is effectively "transparent"

Shorted and open transmission lines are practical and have just about any impedance you want (for a given λ).