Application of the Random Coupling Model to Lossy Ports in Complex Enclosures

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***Abstract*—The random coupling model (RCM) has been successfully demonstrated to characterize the statistics of the impedance matrix for complex multi-port cavities. However, a challenge remains in finding a practical method to measure the necessary parameters that form the model, especially in the case where ports have localized power loss. In this paper, we present practical methods to measure the radiation impedance of the port and the loss parameter of the cavity. In the case of lossy port in the complex cavity, a generalized antenna model is adopted, and the radiation efficiency of the antenna is used to modify RCM's impedance description.**

I. Introduction

The nature of short wavelength radiation inside complex enclosures is of great interested to the EMC community. This has become increasingly important with the prevalence of electronic devices in these enclosures [1]. The complexities of these enclosures make it difficult to solve for the fields deterministically, and thus a statistical approach is used. Wave-chaos is a successful approach that has been used to statistically describe the nature of the fields in complex enclosures [2]. It is based on a combination of the random plane wave approximation, in which the fields at any point in the enclosure is formed by the random superposition of isotropically propagating plane waves with random phases, and Random Matrix Theory to provide statistical distributions. This treatment requires that the enclosure is over-moded and reverberant. The fluctuating field quantities have been extensively studied in wave-chaotic enclosures. The random coupling model is derived from this work. It is a statistical model to characterize the impedance matrix of a multi-port complex electromagnetic cavity [3]–[5].

The main result from the random coupling model [3] is that the random impedance matrix is given in terms of system specific deterministic quantities and universally distributed random variables expressed in the following formula.

$$Z = jX_{rad} + \xi R_{rad}, \tag{1}$$

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where $Z_{rad} = R_{rad}+jX_{rad}$ is the radiation impedance, which is the impedance of the antenna excluding contributions from the cavity. In other words, it is the impedance matrix that would be measured if the cavity walls were moved to infinity. ξ is a complex random variable whose probability distribution is fully characterized by a single loss parameter (α).

A. Loss Parameter

The loss parameter in RCM [3] is the Q-width of a mode of the cavity normalized to the frequency spacing between modes. It is defined as

$$\alpha = \frac{k^2}{\Delta k^2 Q_{comp}} \tag{2}$$

where k is the reference wavenumber of interest. Δk^2 is the mean mode spacing of adjacent eigenvalues near k and is approximated by Weyl's fomula for 3D electromagnetic enclosures in the short wavelength limit as $2\pi^2/kV$ where V is the volume of the cavity. Q_{comp} is the average quality factor of all modes in a given bandwidth, which is given by [6]

$$Q_{comp} = \omega\tau \tag{3}$$

where ω is the angular frequency, and τ is the exponential time constant of the energy decay in the chamber. τ can be computed from inverse Fourier transform of reflection coefficient measurement which we refer to as the band limited impulse response in Fig. 1. Therefore, the loss parameter can be obtained from a simple reflection coefficient measurement at an antenna in enclosure. Other more accurate methods for measuring α have been proposed and tested in [5].

B. Measuring Radiation Impedance

RCM has been successfully demonstrated for 3D electromagnetic enclosures [5], [7]. However, in past works, the enclosures were such that the radiation impedance of the enclosures were easy to measure by lining the boundaries of the enclosure with radiation absorbing material. A practical method to measure the radiation impedance in the case of limited access to enclosure boundaries has yet to be established.

In section II of this paper, we present the time gating technique as a practical, non-intrusive method to measure the

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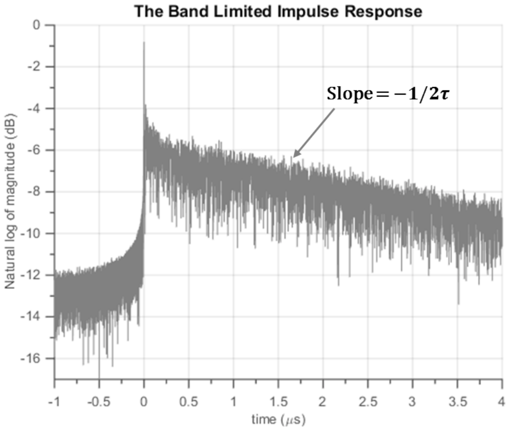

Fig. 1: An example Inverse Fourier transform of the measured reflection coefficient used to compute τ

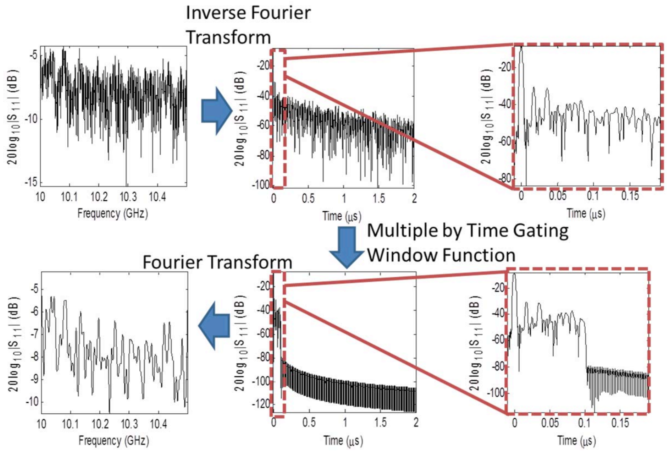

Fig. 2: A example implementation of time gating method

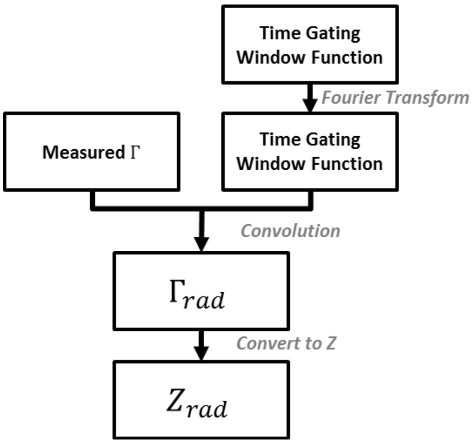

Fig. 3: A flow graph for the time gating method to measure the radiation impedance

radiation impedance that makes it applicable to a broad range of enclosures. Then in section III. we discuss the difficulty of measuring the radiation impedance of lossy ports and present a method to characterize the impedance statistics based on a single measurement of the input impedance at the antenna and the antenna radiation efficiency. In section IV, we describe the implementation of these methods on lossy copper loop trace antenna on FR4 based printed circuit board inside a stainless steel cylindrical chamber and we discuss the results. Finally in section V, we present a summary and discuss the ongoing work.

II. Time Gating Technique

Time gating is a method that selectively removes reflected and delayed time pulses from frequency response measurement by applying a gating window in the time domain. The method is used to improve antenna measurements by removing unwanted reflection from nearby surfaces [8]. Therefore, it is well suited to measure the radiation impedance of our antenna. By time gating the measured reflection coefficient Γ to keep only the early time contribution, the reflections from the cavity are removed. The remaining part of the signal comes from the near field of the antenna, which determines the radiation impedance. This type of time gating has the similar effect to the more intrusive method of covering the boundaries of radiation absorbing material.

In the example case shown in Fig. 2, the frequency domain reflection coefficient measurement from a network analyzer labeled S_{11} is transformed to the time domain. The resulting band limited impulse response is multiplied by a time gating window function to remove part of the response for times greater 0.1 μs. This is transformed back to the frequency domain as the time gated reflection coefficient.

Fig. 3 shows a flow graph of an equivalent method of implementing the time gating technique. The time domain gating window is transformed into the frequency domain and convolved with the reflection coefficient. Here, we have used the identity that multiplication in the time domain is equivalent to a convolution in the frequency domain, similar to some network analyzers [9]. Finally the time gated reflection coefficient is converted to radiation impedance using equation 14. Therefore, this is a practical nonintrusive method to measure the radiation impedance. However, if there is significant power loss at the port, measurement of the radiation impedance will be difficult and the characterizing statistics of the impedance will require additional information, as described in the next section.

III. Lossy Antennas

Antennas with significant insertion loss make it difficult to measure the radiation impedance. Most antennas are designed to have very low insertion loss. For this type of antenna, the simple model shown in Fig. 4, we can measure the radiation impedance in situ by the time gating method described in section II. A single port network analyzer measurement at the antenna over the frequencies of interest can capture the radiation impedance. Using the radiation impedance and the loss parameter, the impedance statistics at that antenna can be fully characterized.

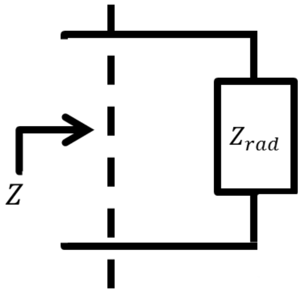

Fig. 4: A model for antenna with negligible loss

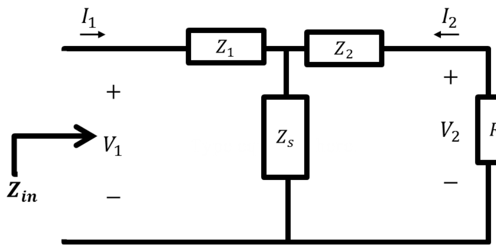

Fig. 5: A generalized model used for the impedance analysis of a lossy antenna. R is the radiation resistance and the radiation reactance is lumped with Z_2

However, there are other antennas that are not designed or intended to couple electromagnetic energy, which are also of interest to the EMC community [10]. Electromagnetic interference is often caused by strong electromagnetic energy coupling through unintended ports. In the most general case, antennas can be modeled with "T-network" shown in Fig. 5, where Z_1, Z_2, and Z_s are arbitrary complex impedances. Thus, to characterize the antenna, three complex impedances must be determined. The loss can be modeled as a resistive element in any of the branches of the T-network. If the antenna has significant insertion loss the radiation impedance cannot be measured by the time gating method or by lining the cavity walls with absorbers. In this work, we consider the case when such a lossy antenna is located in an enclosure where the condition for random coupling model applies. That is, the wavelength is large compared to the reverberant enclosure. In these enclosures, we often only have measurement access to Z_{in}, but we can't measure the addition impedance in the T-network. In this section, the impedance statistics are described for a single port enclosure such that the only additional information required is the radiation efficiency of the antenna.

The main result presented in this section is the following equation for the impedance of a lossy antenna inside a complex enclosure.

$$Z_{in} = Z_{ant} + \eta Re\{Z_{ant}\}\delta\xi \tag{4}$$

where η is the radiation efficiency of the antenna. $\delta\xi$ is the $\xi - 1$ where ξ is the fluctuating normalized impedance as described in Eq. 1. Z_{ant} is the input impedance of the lossy antenna radiating in free space and will be defined in the derivation that follows.

For the following analysis, a two port model of the antenna shown in Fig. 5 is adopted. The elements of the two port impedance matrix are

$$Z_{11} = Z_1 + Z_s \tag{5}$$

$$Z_{22} = Z_2 + Z_s \tag{6}$$

$$Z_{12} = Z_{21} = Z_s. \tag{7}$$

When the antenna is placed in free space with no reflection, the measured input impedance is given as

$$Z_{in} = Z_{11} - \frac{Z_s^2}{Z_{22} + R} \equiv Z_{ant} \tag{8}$$

The radiation efficiency of this antenna is defined as the ratio of the power radiated to the input power

$$\eta = \frac{P_R}{P_{in}} \tag{9}$$

where $P_R = R|I_2|^2$ and $P_{in} = Re\{Z_{in}\}|I_1|^2$. Solving the circuit, we obtain the following expression for the radiation efficiency,

$$\eta = \frac{R}{Re\{Z_{ant}\}} \left| \frac{Z_s^2}{R + Z_{22}} \right|^2 \tag{10}$$

If this antenna is placed in a complex enclosure, we replace R with $R(1 + \delta\xi)$ in equation 8. For a typical high loss cavity where $\alpha \gg 1$ and $|\delta\xi| \ll 1$, the input impedance can approximated as follows,

$$Z_{in} \approx \left(Z_{11} - \frac{Z_{12}^2}{Z_{22} + R} \right) + \left(\frac{RZ_{12}^2}{(Z_{22} + R)^2} \right) \delta\xi \tag{11}$$

Factoring out the phase from the second term, we obtain

$$Z_{in} = \left(Z_{11} - \frac{Z_{12}^2}{Z_{22} + R} \right) + \left| \frac{RZ_{12}^2}{(Z_{22} + R)^2} \right| \delta\xi\prime \tag{12}$$

where $\delta\xi\prime = \delta\xi e^{j\phi}$ and ϕ is the phase of $RZ_{12}^2/(Z_{22} + R)^2$.

Since $\delta\xi$ is approximated by a complex gaussian random variable with independent and identically distributed real and imaginary parts, the phase of $\delta\xi$ will be uniformly distributed. This means that $\delta\xi$ and $\delta\xi\prime$ have the same statistical properties. Therefore we can drop the prime from $\delta\xi$. Now comparing Eq. 12 with Eq. 10, we arrive at the expression for the impedance given in Eq. 4 with quantities that can be easily measured.

IV. Results

To demonstrate these methods and for specificity, we focus on a copper loop antenna printed on a lossy FR4 based circuit board (Fig. 7). We measured this antenna inside a stainless steel cylindrical enclosure (6) for two sets of frequency. The loss parameters, measured according to section I-A, are 1.6 and 24.1 for the frequency ranges 1.5GHz to 3GHz and 9GHz to 11GHz, respectively.

The FR4 material has low dielectric loss in the lower frequency range, but the loss becomes significant at these higher frequencies, which reduces the radiation efficiency.

Fig. 6: The cylindrical stainless steel enclosure with a rotating mode stirrer

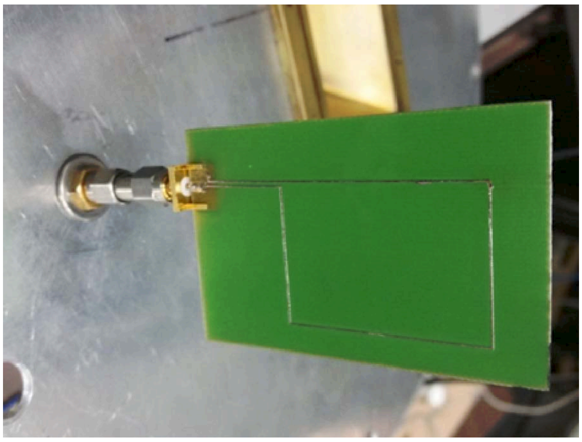

Fig. 7: The copper loop trace on FR4 PCB used in the experiments

A. *Radiation Efficiency*

In order to measure the radiation efficiency, we modeled the material in ANSOFT High Frequency Structural Simulator (HFSS). Fig. 8 shows the HFSS model for the antenna. A radiation boundary is assigned to the large box surrounding the antenna. The radiation efficiency is computed as the ratio of the power radiated through the radiation boundary to the power incident at the input port of the antenna. Fig. 9 and Fig. 10 show the resulting radiation efficiency computed for the two frequency ranges. The average radiation efficiency is computed to be 0.56 and 0.81 for the frequency ranges 1.5GHz to 3GHz and 9GHz to 11GHz, respectively. Using these radiation efficiency values, we can characterize the impedance statistics using Eq. 4.

B. *Comparing Normalized Impedance Distributions*

Impedance measurements of this antenna have been conducted assuming the general antenna model in Fig. 5. This is done by directly measuring the reflection coefficient (S_{11}) using a vector network analyzer for 32001 points over the frequency range. This measurement is repeated for 50 different

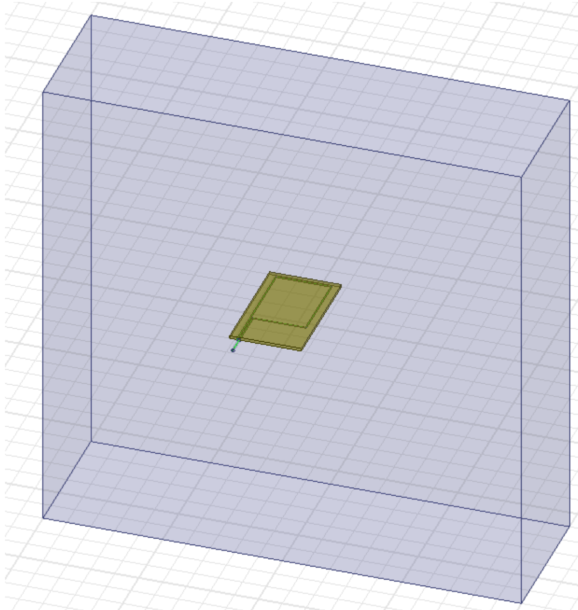

Fig. 8: A copper loop trace on FR4 PCB modeled in HFSS inside a radiation box

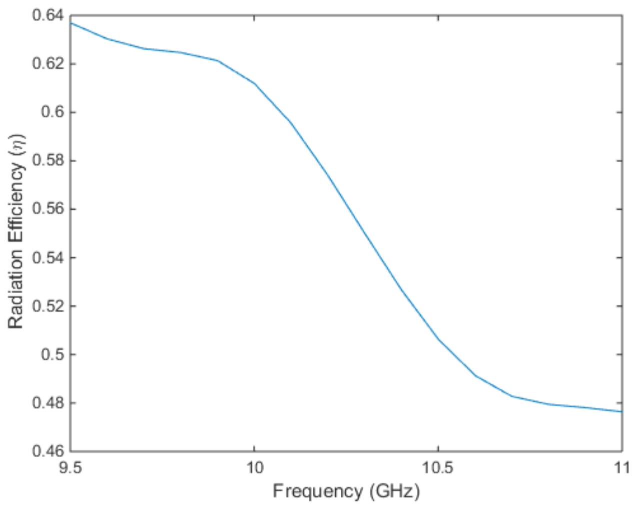

Fig. 9: The radiation efficiency of the loop trace on antenna computed in HFSS for 9.5GHz - 11GHz

positions of a mode stirrer inside the enclosure. The normalized impedance is computed for each measurement data point using Eq. 13, where the HFSS computed radiation efficiency is applied.

$$\delta\xi_M = \frac{Z_{in} - Z_{ant}}{\eta Re\{Z_{ant}\}} \tag{13}$$

Z_{in} is generated from the measured reflection coefficients, Eq. 14.

$$Z_{in} = Z_0^{\frac{1}{2}}(I - S_{11})^{-1}(I + S_{11})Z_0^{\frac{1}{2}} \tag{14}$$

Z_{ant} is computed from Z_{in} using the time gating method described is section II.

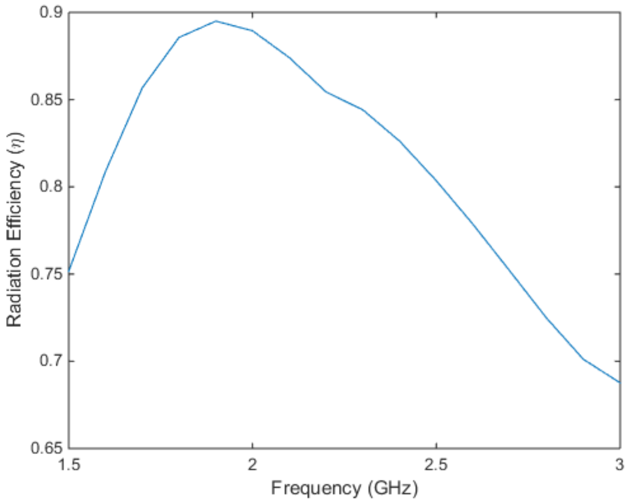

Fig. 10: The radiation efficiency of the loop trace antenna computed in HFSS for 1.5GHz - 3GHz

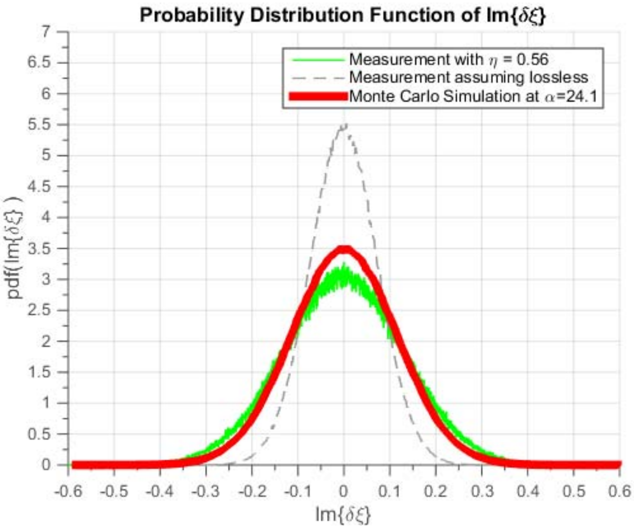

Fig. 12: Comparison of the probability function of the imaginary part of the normalized impedance for 9.5GHz - 11GHz

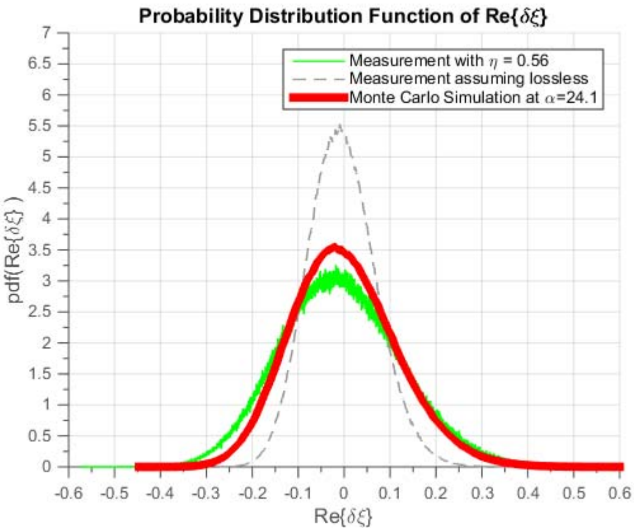

Fig. 11: Comparison of the probability function of the real part of the normalized impedance for 9.5GHz - 11GHz

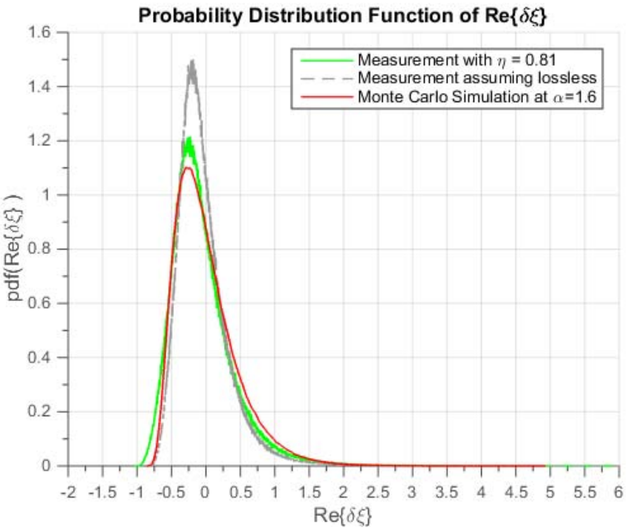

Fig. 13: Comparison of the probability function of the real part of the normalized impedance for 1.5GHz - 3GHz

The probability distribution function (pdf) of the real and imaginary parts of the normalized impedance is generated from a histogram of the large ensemble of measurements. Figures 11 to 14 show the pdfs are in good agreement with the predicted pdfs. The predicted pdfs are obtained from the random matrix based Monte Carlo simulation [7], where the only parameter required for the Monte Carlo simulations is the measured loss parameter α. This agreement also shows the significance of this new model. That is, if we had assumed the simple antenna model in Fig. 4, the pdfs, shown in grey in the Figures 11 to 14, would have been incorrect.

V. Conclusion

We have shown that for a lossy antenna in a lossy cavity, the coupling statistics can still be characterized. Based on the description from the random coupling model and the use of the radiation efficiency, the probability distribution function of the impedance matrix can be fully characterized. Furthermore, practical measurement techniques, that are of interest to EMC community are demonstrated a typical real world enclosure. We are currently expanded this method to a two ports system and validating it with experimental results.

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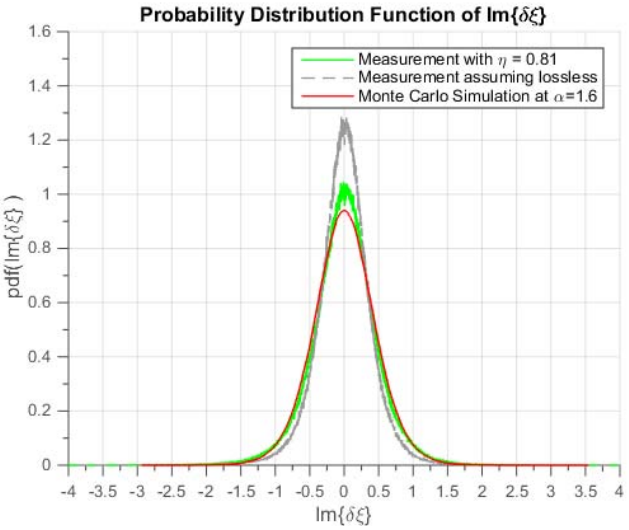

Fig. 14: Comparison of the probability function of the imaginary part of the normalized impedance for 1.5GHz - 3GHz

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