Summarizing the different methods to obtain the cavity-loss parameter α

[Refer Hemmady Thesis for further details]

Method 1: Direct estimation of α from first principles: $\alpha = k^2/(\Delta k_n^2 Q)$

Procedure: **[See section 4.1.2 and appendix C of Hemmady Thesis]**

i) Determine : $k = 2\pi f / c$

ii) Estimate cavity Q (see appendix C)

iii) Determine : $\Delta k_n^2 \approx 4\pi / A \quad (2D) \quad or \quad \Delta k_n^2 \approx 2\pi^2/(kV) \quad (3D)$

iv) Determine: $\alpha = k^2/(\Delta k_n^2 Q)$

Advantage:

i) Fast and easy to determine

ii) Useful as a quick estimate for α if cavity-losses are low and cavity resonances are not overlapping

Disadvantage:

i) Difficult to estimate the average Q of the resonator modes especially if the cavity-resonances overlap (high loss limit)

ii) For low frequencies, contributions from the lower order corrections due to the perimeter of the cavity must be taken into account when determining the value of Δk_n^2 in the quasi-two-dimensional cavity. Similar corrections are necessary in three-dimensions.

Method 2:

Comparing the PDFs of Re[z] and Im[z] obtained from measurements with corresponding PDFs numerically generated from RMT using α as a fitting parameter.

Procedure: **[See section 4.1.3 of Hemmady Thesis]**

i) Numerically generate PDFs of Re[z] and Im[z] using Random Matrix Monte Carlo simulations for different values of α.

ii) Experimentally determine the PDFs of Re[z] and Im[z] for a given frequency range (where α is approximately constant).

iii) Choose that value of α which minimizes the error between the PDFs in (i) and (ii).

Advantage:

i) More accurate than method-1

ii) Possible to estimate the error in determining α.

iii) Valid for lossy cavities with overlapping resonances.

Disadvantage:

i) Computationally intensive and time-consuming

ii) Need to numerically generate large ensembles of normalized impedance values using Random Matrix Monte Carlo simulations for a range of α-parameter values in order to perform the fitting procedure. Prone to statistical errors which could arise due to poor statistics (insufficient number of samples) of the numerically generated RMT z PDFs.

Method 3:

Estimating the loss-parameter for the data-set from the relation between the variance of Re[z] or Im[z] PDFs and α obtained from Random Matrix Monte Carlo simulations.

Procedure: **[See introduction to Chapter 5 of Hemmady Thesis]**

(i) Numerically generate PDFs of Re[z] and Im[z] using Random Matrix Monte Carlo simulations for different values of α.

(ii) Fit the relation between the variance of the numerically-generated Re[z] or Im[z] PDFs and α to a polynomial (in α) of high order.

(iii) Determine the variance of experimentally measured Re[z] or Im[z] PDFs.

(iv) Solve the polynomial in (ii) to estimate the value of α for the experimental data-set.

Advantage:

i) More accurate than method-1

ii) Possible to estimate the error in determining α.

iii) Valid for lossy cavities with overlapping resonances.

Disadvantage:

i) Computationally intensive and time-consuming

ii) Need to numerically generate large ensembles of normalized impedance values using Random Matrix Monte Carlo simulations for a range of α-parameter values in order to perform the fitting procedure. Prone to statistical errors which could arise due to poor statistics(insufficient number of samples) of the numerically generated RMT z PDFs.

iii) Errors in the experimental calibration or imperfect impedance normalization can lead to systematic errors due to outlying data points, which give an erroneous value for the estimated variance of Re[z] or Im[z] PDFs. This will affect the determined α.

Method 4:

Estimating the loss-parameter for the data-set from the relation between the variance of Re[z] or Im[z] PDFs and α obtained from the Random Coupling Model.

Procedure: **[See section 4.1.3 of Hemmady Thesis]**

(i) Determine the variance of experimentally measured Re[z] or Im[z] PDFs.

(ii) Determine: $\alpha = \frac{1}{\pi\sigma^2_{\mathrm{Re}[z]}} = \frac{1}{\pi\sigma^2_{\mathrm{Im}[z]}}$

Advantage:

i) Fast and easy to determine

ii) Useful as a quick estimate for α.

iii) Valid for lossy cavities with overlapping resonances.

Disadvantage:

i) Valid only when $\alpha > 5$

ii) Errors in the experimental calibration or imperfect impedance normalization can lead systematic errors due to outlying data points, which give an erroneous value for the estimated variance of Re[z](or Im[z]). This will affect the determined α.

iii) Valid only for TRS systems. For BTRS systems use Eq.(2.5)

Method 5:

Estimating the loss-parameter for the data-set from the relation between the dephasing parameter (γ) and <T>, where <T>=1- <eigenvalues of $\overleftrightarrow{s}\overleftrightarrow{s}^{\dagger}$>

Procedure: **[See section 5.3.3 of Hemmady Thesis]**

i) Determine <T> from eigenvalues of $\overleftrightarrow{s}\overleftrightarrow{s}^{\dagger}$, where $\overleftrightarrow{s}$ is the normalized scattering matrix.

ii) Determine γ from <T> and Eq.(5.4)

iii) Determine: $\alpha=\gamma/4\pi$

Advantage:

i) More accurate than methods-(1,4); Works for any arbitrary number of driving ports.

ii) Possible to estimate the error in determining α.

iii) Valid for lossy cavities with overlapping resonances.

iv) Possible to determine α from the normalized scattering matrix eigenvalues.

Disadvantage:

i) Errors in the experimental calibration or imperfect impedance normalization can lead systematic errors due to outlying data points, which give an erroneous value for the estimated <T>. This will affect the determined α.

Method 6:

Estimating the loss-parameter for the data-set from the impedance-based Hauser-Feshbach relation.

Procedure: **[See Chapter 7 of Hemmady Thesis]**

(i) Determine the Hauser-Feshbach impedance ratio from the measured raw cavity impedance matrix.

(ii) Estimate the value of α from Eq.(7.2)

Advantage:

i) Fast and easy to determine

ii) α can be determined from the raw cavity data - no need to normalize the cavity data using the radiation impedance

Disadvantage:

Not been experimentally established as yet !

i) Practically useful only when $\alpha < 1$

ii) Not applicable to one-port systems

iii) Errors in the experimental calibration can lead to systematic errors due to outlying data points, which give an erroneous value for the estimated variance of impedance matrix elements. This will affect the determined α.

iv) Presence of short-ray orbits in the cavity leads to fluctuations in the Hauser-Feshbach impedance ratio. Thus a statistical average of the impedance ratio as a function of frequency must be included.